Math 120 Merit Workshop
Worksheet 26

1. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

2. Suppose you are to make a rectangular box with a square base from two different materials. The material for the base costs \$2 per square foot, the material for the rest costs \$1 per square foot. Find the dimensions of the box of the greatest volume if you are allowed to spend \$144 on the materials to make it.

3. Let $C(x) = 2\sqrt{x} + \frac{x^2}{8000}$ be a cost function
   a) Find the cost, average cost, and marginal cost at a production level of 1000 units

   b) Find the production level that will minimize the average cost

   c) Find the minimum average cost.

4. The cost, in dollars, of producing x yards of a certain fabric is $C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$ and the company has found that if it sells x yards, it can charge $p(x) = 29 - 0.00021x$ dollars per yard of fabric. Find the production level that will maximize profit.

5. An aircraft manufacturer wants to determine the best selling price for a new airplane. The company estimates that the initial cost of designing the airplane and setting up the factories in which to build it will be 500 million dollars. The additional cost of manufacturing each plane is given by the function $m(x) = 20x - 5x^{3/4} + 0.01x^2$, where x is the number of aircraft produced and m is the manufacturing cost, in millions of dollars. The company estimates that if it charges a price p (in millions of dollars) for each plane, it will be able to sell $x(p) = 320 - 7.7p$ planes.
   a) Find the cost, demand, and revenue functions.

   b) Find the production level and associated selling price of the aircraft that maximizes profit.

6. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$400 per month. A market survey suggests that, on the average, one additional unit will remain vacant for each \$5 increase in rent. What rent should the manager charge to maximize revenue

7. Discuss the general formula for Newton's Method (see Section 4.9). Do the first four iterations of Newton's Method to find a root of the equation $x^3 + 2x^2 + x + 1 = 0$

8. Give a *function* whose *derivative* is the following.

a) $3x^2$

b) $3\cos(3x)$

c) $5xe^{5x^2}$

d) $3x^2 + 10x + 3$

10. A steel pipe is being carried down a hallway 9ft wide. At the end of the hallway is a right-angle turn into a hallway 6ft wide. The pipe must be carried horizontally. The length of the pipe is limited by the walls as it spins around the corner

a) If the pipe makes an angle of θ with the wall of the hallway, then how long could the pipe be? (Use your trigonometry, and split the pipe-hence the problem-into two pieces, one in each hallway.)

b) You should now have the length of the pipe in terms of the angle ($L = f(\theta)$). What is the length of the longest pipe that can be carried around the corner?

c) Does this length correspond to a max or min point of $f(\theta)$? Confirm your intuition with calculus.