

Math 120 Merit Workshop  
Worksheet 8

1. Use the Intermediate Value Theorem to show that the equation  $\sqrt{x+5} = \frac{1}{x+3}$  has a solution in the interval  $(-2.5, -1)$ .

2. Use what you know about asymptotes and intercepts to carefully sketch the graphs of the following functions:

a)  $y = \frac{1}{x-9}$

b)  $y = \frac{x}{x-9}$

c)  $y = \frac{1}{x^2-9}$

d)  $y = (1-x)(x-3)^2(x-5)^3$

3. Find the limit:

a)  $\lim_{x \rightarrow \infty} \frac{5x^3 - x^2 + 2}{2x^3 + x - 3}$

b)  $\lim_{r \rightarrow \infty} \frac{r^4 - r^2 + 1}{r^5 + r^3 - r}$

4. Sketch a graph that satisfies all of the following conditions

a)  $\lim_{x \rightarrow +\infty} f(x) = 2$  and  $\lim_{x \rightarrow -\infty} f(x) = -2$

b)  $f(x)$  has a horizontal asymptote at  $y = -2$  and  $|f(x)| \leq 4$  for all  $x$

c)  $f(x)$  is odd and has asymptotes at  $y = 1$ ,  $x = -4$ ,  $x = 4$

d)  $\lim_{x \rightarrow -2} = \infty$ ,  $\lim_{x \rightarrow -\infty} = 3$ ,  $\lim_{x \rightarrow \infty} = -3$

e)  $\lim_{x \rightarrow \pm\infty} = 0$ ,  $\lim_{x \rightarrow 0} = -\infty$ ,  $\lim_{x \rightarrow 3^-} = \infty$ ,  $\lim_{x \rightarrow 3^+} = -\infty$ ,  $f(2) = 0$

5. Is there a number  $a$  such that  $\lim_{x \rightarrow -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$  exists? If so, find the value of  $a$  and the value of the limit.

6. Find all values of  $x$  in  $[0, 2\pi]$  that satisfy each equation:

a)  $\sin 2x = \cos x$

b)  $\sin x = \tan x$

7. Particle Man is wasting away his afternoon (doing the only thing a particle can) by tracing out the curve  $y = \sqrt{x}$ . When he gets to  $x = 4$ , he wonders "If I had just traced out the straight line between here and where I started at  $x = 0$ , I'd have walked a line with a slope of..."

a) Finish Particle Man's sentence for him.

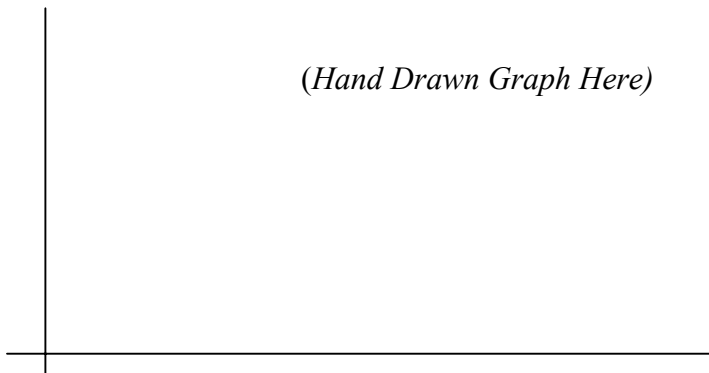
- b) Now Particle Man is really walking on edge! (Ha Ha) He thinks that he can figure out exactly the slope of the tangent line to  $y = \sqrt{x}$  at  $x = 4$ . The problem is particles aren't really that bright. Can you do it for him?
- c) Finally Particle Man wonders if he can figure out what the slope of the tangent line would be at any point  $x = a$ . Help him out one more time.

8. Find the equation of the tangent line to the curve at the given point

a)  $y = 1 - 2x - 3x^2$ ,  $(2, 7)$

b)  $y = \frac{2}{(x+3)}$ ,  $(a, \frac{2}{a+3})$

9. The graph shows the position function of a car with respect to time:



- a) How is the velocity of the car represented on this graph?
- b) Where is the velocity equal to zero?
- c) Where is the velocity constant?
- d) What is the initial velocity?
- e) Put the letters A, B, C, D, E in order of the smallest velocity to the largest velocity.

10. The displacement (in meters) of a particle moving in a straight line is given by the equation of motion  $s = 4t^3 + 6t + 2$ , where  $t$  is measured in seconds. Find the velocity of the particle at times  $t = a$ ,  $t = 1$ ,  $t = 2$ , and  $t = 3$ .