

Some combinatorial principles about linear orderings weaker than Ramsey's Theorem

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We analyze degree theoretically and in terms of reverse mathematics some combinatorial principles about linear orderings such as (ASDS): Every infinite linear ordering has an infinite ascending sequence or an infinite descending one. All of the principles follow from Ramsey's theorem for pairs but none from WKL_0 (over RCA_0). In particular, we consider two, $ASDS^*$ and $ASDS^\#$ (analogous to, and consequences of, SRT_2^2 and COH, respectively) which are incomparable over RCA_0 but which join up to ASDS in the sense that $RCA_0 \vdash ASDS \leftrightarrow (ASDS^* \ \& \ ASDS^\#)$ while $(RCA_0 \ \& \ ASDS^*) \not\vdash ASDS^\#$ and $(RCA_0 \ \& \ ASDS^\#) \not\vdash ASDS^*$. Our reverse mathematical conclusions follow from degree theoretic analyses, forcing arguments and conservativity results. The Σ_2 bounding scheme in particular plays an interesting role.