Computable Vaughtian Model Theory

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In 1961 Robert Vaught studied countable models of a complete theory T, and introduced prime, homogeneous, and saturated models, defined in terms of types in the Stone space $S(T)$. Roughly simultaneously, Gerald Sacks, Tony Martin, and Alistair Lachlan classified (Turing) degrees intermediate between $\mathbf{0}'$ (the halting problem) and $\mathbf{0}$ in terms of their information content being *high* or *low*, and they related the corresponding classes $\mathbf{H_n}$ and $\mathbf{L_n}$, $n \in \omega$, to algebraic properties in various structures. A decade later Harrington and Goncharov-Nurtazin helped launch computable model theory by proving that a complete atomic decidable (CAD) theory T has a decidable prime model iff there is a computable listing of the principal types in $S(T)$.

In this talk we describe recent and ongoing research on five projects concerning countable models of a complete decidable (CD) theory T: (1) degrees of prime models; (2) degrees bounding prime models; (3) degrees bounding homogeneous models; (4) degrees of isomorphic copies of special homogeneous models related to the Goncharov-Peretyatkin effective extension property (EEP); and (5) degrees bounding saturated models of T, if T has types all computable (TAC). Of particular interest are the degrees $\mathbf{H_n}$ and $\mathbf{L_n}$ in the high/low hierarchy, and the degrees of complete extensions of Peano arithmetic. These projects were conceived and directed by Soare and Hirschfeldt at the University of Chicago and carried out by them along with coworkers and students as we shall explain.

*This lecture is dedicated to the late Robert L. Vaught and to Carl G. Jockusch, Jr. for the elegance, innovation, and clarity they brought to model theory and computability theory, respectively. Their research directly affects all five projects.