Logic and Mathematics:
connections and interactions

May 21–25, 2003

ABSTRACTS OF INVITED TALKS

Mathematics Department
University of Illinois
Urbana, Illinois 61801 USA

Wednesday, May 21,2003; 11:00–noon

Orbit spaces in $\delta-$geometry and analytic uniformization

Alexandru Buium
University of New Mexico

Abstract: Let X be an algebraic curve, let Y be an algebraic curve in the product $X \times X$, and let E be the smallest equivalence relation in $X \times X$, containing Y. Then, in most interesting cases, E is a countable infinite union of distinct curves in $X \times X$ so the categorical quotient X/E in the category of varieties reduces to a point. There is a natural way to enlarge algebraic geometry, however, by adjoining operators δ acting on the ground field or ring. Results and conjectures will be presented to the effect that X/E ceases to reduce to a point in "almost all" these "δ-geometries" if and only if the pair (X, Y) admits a complex analytic uniformization. The latter condition holds for instance if X is the projective line and Y defines a "spherical" correspondence, a flat dynamical system, or a Hecke correspondence.

Friday, May 23, 2003; 2:00–3:00

Finite groups and groups of finite Morley rank

Gregory Cherlin
Rutgers University

Abstract: It has been conjectured that simple groups of finite Morley rank are algebraic (Chevalley groups over algebraically closed fields). The Borovik program seeks to determine the possible 2-Sylow structure in a hypothetical minimal counterexample, with (decreasing) emphasis on the so-called "tame" case. This is a task to which a significant part of the technology used to classify the finite simple groups is well adapted. The ideal theorem would be that the "2-rank" is necessarily quite small.

Recently progress depends on improvements in the finite Morley rank analogs of two chapters of finite group theory: "signalizer functor" theory and the classification of groups with "strongly embedded" subgroups. A critical ingredient in the latter case is a powerful model theoretic result of Frank Wagner on fields of finite Morley rank.

In purely technical terms, the problem is (1) to prove (or rather, to avoid having to prove) a "solvable signalizer functor theorem" and (2) to classify groups of even type with weakly embedded subgroups under a mild inductive hypothesis, weaker than K*. I aim to explain what all of this actually means, and why it makes a difference.

Reference: Simple groups of finite Morley rank 2002 (a survey), a preprint available at `http://www.math.rutgers.edu/~cherlin/FMR`.

Wednesday, May 21, 2003; 2:00–3:00

Modeling functions: transseries and analyzability

Ovidiu Costin
Rutgers University

Abstract: The talk focuses on recent advances in the representation of functions of natural origin by combinations of exponentials, powers and logs (transseries), its implications and some applications, with a view on the formal and analytical aspects.

Friday, May 23, 2003; 9:30–10:30

Stochastic fluid mechanics, attractors and nonstandard analysis

Nigel Cutland
University of Hull

Abstract: Over the past 12 years a variety of new results in theoretical stochastic fluid mechanics have been obtained (in collaboration with Marek Capinski and Jerry Keisler) using nonstandard analysis - particularly Loeb measure techniques. The most recent work is on attractors and involves the Fajardo/Keisler neometric space machinery.

This talk will survey these results with emphasis on aspects where the use of NSA/Loeb measures (or some equivalent such as saturated adapted probability spaces) seems to be essential. For example, the strongest formulation of a new result about existence of attractors for 3-D stochastic Navier-Stokes equations involves the notion of neo-compactness.

The talk will not assume any prior knowledge of fluid mechanics and only the rudiments of NSA.

Thursday, May 22, 2003; 2:00–3:00

Betti numbers of definable sets

Andrei Gabrielov
Purdue University

Abstract: A spectral sequence associated with a surjective continuous map provides upper bounds on the Betti numbers of a set defined by an expression with quantifiers, in terms of the Betti numbers of some auxiliary sets defined by quantifier-free formulas. Applications include sub-Pfaffian and "sub-algebraic" sets. For a "limit set" defined by the relative closure operation on a semi-Pfaffian family (in particular, for a Hausdorff limit of a family of semi-Pfaffian sets) Betti numbers can be estimated in terms of the Betti numbers of auxiliary semi-Pfaffian sets associated with the fibers of the family.

Saturday, May 24, 2003; 9:30–10:30

Fraïssé limits, Ramsey theory, and topological dynamics of automorphism groups

Alexander S. Kechris
California Institute of Technology

Abstract: (joint work with V. Pestov and S. Todorcevic) It has been recently discovered that the study of topological dynamics of automorphism groups, particularly concerning extreme amenability and the calculation of universal minimal flows, is closely connected to the Fraïssé theory of amalgamation and ultrahomogeneity, and the Ramsey theory for classes of finite structures. In this talk, I will explain these connections and discuss applications to the topological dynamics of automorphism groups.

Wednesday, May 21, 2003; 3:30–4:30

Vandermonde matrices in model theory, complexity theory and elementary geometry

Pascal Koiran
Ecole Normale Supérieure, Lyon

Abstract: A Liouville function is the sum of a series of the form $\sum_{n\geq 1} x^n/a_n$ where the a_n form a "very fast growing" sequence of integers. In this talk I will exhibit the complete first-order theory of the complex field field expanded with a Liouville function. This is probably the first example of a complex analytic function whose model theory is well understood. I will try to highlight the role played by Vandermonde matrices, and their applications to elementary geometry and NP-completeness.

Friday, May 23, 2003; 3:30–4:30

Constructible motivic functions, integrals with parameters, and cell decomposition

Francois Loeser
Ecole Normale Supérieure, Paris

Abstract: We will present a new framework for motivic integration based on cell decomposition. A basic role is played by constructible motivic functions and their direct images by definable morphisms. We shall present some of the main applications of the theory including a general structure theorem for motivic integrals depending on parameters and a motivic version of Fubini's theorem. This is joint work with Raf Cluckers.

Thursday, May 22, 2003; 9:30–10:30

Ramsey theory and Banach spaces

Edward W. Odell
University of Texas, Austin

Abstract: Ramsey theory has had a significant impact on the study of the geometry of Banach spaces over the past 30 years. Applications include the theory of spreading models, a key lemma in Rosenthal's famous ℓ_1 theorem, partial unconditionality, distortion problems, Gowers block Ramsey theorem and numerous other results. We will present a survey of these applications. This talk is aimed at the non-expert.

Wednesday, May 21, 2003; 9:40–10:40

Hilbert's Tenth Problem and number-theoretic generalizations

Bjorn Poonen
University of California, Berkeley

Abstract: Hilbert's Tenth Problem was to find an algorithm (Turing machine) to decide, given a multivariable polynomial equation with coefficients in $\mathbb{Z}$, whether it has a solution with coordinates in $\mathbb{Z}$. Around 1970, Matijasevic completed a proof that no such algorithm exists. On the other hand, replacing $\mathbb{Z}$ by $\mathbb{Q}$ in both places results in a question whose answer is not known: an equivalent form of the question is, is there an algorithm for deciding whether a variety over $\mathbb{Q}$ has a rational point? Also, if one replaces $\mathbb{Z}$ by the ring of integers of a number field k, the existence of an algorithm is known only for special k. I will present a survey of some recent progress on these problems.

Saturday, May 24, 2003; 3:30–4:30

Trivial observations about the André-Oort conjecture

Thomas Scanlon
University of California, Berkeley

Abstract: Y. André and F. Oort have raised a conjecture about subvarieties of Shimura varieties containing a dense set of special points in analogy with the Manin-Mumford conjecture for abelian varieties. The model theoretic methods applied to this latter problem are not strictly relevant as the corresponding sets defined in difference fields have trivial forking. However, the model theory of difference fields and general stability theory still have some consequences for the André-Oort conjecture. We describe some of these consequences, including uniformity and p-adic fibred version for subvarieties of universal abelian schemes.

Sunday, May 25, 2003; 9:30–10:30

Diophantine geometry over groups and the elementary theory of free and hyperbolic groups

Zlil Sela
Hebrew University

Abstract: We study sets of solutions to equations over a free group, projections of such sets, and the structure of elementary sets defined over a free group. The structure theory we obtain enables us to answer some questions of A. Tarski, and to classify those finitely generated groups that are elementarily equivalent to a free group. Connections with low dimensional topology, a generalization to (Gromov) hyperbolic groups, stability, and further aspects of the first order theory of free (and hyperbolic) groups will also be discussed.

Friday, May 23, 2003; 11:00-noon

Asymptotic cones and questions of Margulis and Gromov

Katrin Tent
University of Würzburg

Abstract: (joint work with L. Kramer) Using ultrapowers of Lie groups and nonstandard techniques, we outline a short and algebraic proof of the Margulis conjecture, which states that any quasi-isometry between Riemannian symmetric spaces induces a proper isometry. We obtain an explicit description of the asymptotic cone of a symmetric space which allows us to answer a question of Gromov on asymptotic cones of finitely presented groups.

Saturday, May 24, 2003; 11:00–noon

Superrigidity and the classification problem for the torsion-free abelian groups of finite rank

Simon Thomas
Rutgers University

Abstract: In 1937, Baer solved the classification problem for the torsion-free abelian groups of rank 1. Since then, despite the efforts of such mathematicians as Kurosh and Malcev, no satisfactory solution has been found for the classification problem for the torsion-free abelian groups of rank $n \geq 2$. So it is natural to ask whether the classification problem is genuinely more difficult for the groups of rank $n \geq 2$. In this talk, I will explain how this question can be answered, using Zimmer's superrigidity theorem for actions of lattices in higher rank semisimple Lie groups.

Thursday, May 22, 2003; 3:30–4:30

Universality and randomness: results and conjectures

Anatoly Vershik
Steklov Institute of Mathematics, St. Petersburg

Abstract: There are several categories in which a unique universal homogeneous object exists. The most known example, which is very popular in the theory of models, is the universal graph (Rado). It is much less known that the analogous situation takes place with Polish metric spaces. Such an object had been defined in 1924 by P. Urysohn. At the same time, the random graph in the sense of Erdös-Renji (and in a more general sense) is also with probability 1 a universal graph. It happens that for metric spaces we also can define a meaning of "random metric space" and prove that such a space with probability 1 is the Urysohn space. We developed some tools (matrix method) for studying such problems.

Thursday, May 22, 2003; 11:00–noon

A proper o-minimal expansion of the additive group of rationals

Alex Wilkie
University of Oxford

Abstract: We settle, negatively, a conjecture of Steinhorn. In fact we show that any countable elementary substructure of any countable reduct of $\mathbb{R}_{an}$ can be represented, o-minimally, on the unit interval in the additive group of rationals.

Saturday, May 24, 2003; 2:00–3:00

A question of Urysohn

Martin Ziegler
University of Freiburg

Abstract: In 1927 Urysohn constructed a separable metric space U with the following two properties:
• (universality) every separable metric space is embeddable in U,
• (homogeneity) every isomorphism between finite subspaces extends to an automorphism of U;
and asked if U is the only separable metric space with these properties. We give a negative answer to his question. We also show:

Theorem 1. *Urysohn's space is characterized by each of the following conditions:*
- *U is separable, universal, homogeneous and complete;*
- *U is separable, universal, and strongly homogeneous.*

In the above, *strongly homogeneous* means that every isomorphism between totally bounded subspaces extends to an automorphism.

Sunday, May 25, 2003; 2:00–3:00

Analytic Zariski structures

Boris Zilber
University of Oxford

Abstract: Zariski structures introduced by Hrushovski and the author proved to be a very useful concept in some applications found by Hrushovski. This talk will discuss an extension of the notion designed to deal with essentially transcendental analytic or 'pseudo-analytic' objects.