

Betti numbers of definable sets

Andrei Gabrielov
Purdue University

Abstract: A spectral sequence associated with a surjective continuous map provides upper bounds on the Betti numbers of a set defined by an expression with quantifiers, in terms of the Betti numbers of some auxiliary sets defined by quantifier-free formulas. Applications include sub-Pfaffian and “sub-algebraic” sets. For a “limit set” defined by the relative closure operation on a semi-Pfaffian family (in particular, for a Hausdorff limit of a family of semi-Pfaffian sets) Betti numbers can be estimated in terms of the Betti numbers of auxiliary semi-Pfaffian sets associated with the fibers of the family.