Hilbert's Tenth Problem and number-theoretic generalizations

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Abstract: Hilbert's Tenth Problem was to find an algorithm (Turing machine) to decide, given a multivariable polynomial equation with coefficients in $\mathbb{Z}$, whether it has a solution with coordinates in $\mathbb{Z}$. Around 1970, Matijasevic completed a proof that no such algorithm exists. On the other hand, replacing $\mathbb{Z}$ by $\mathbb{Q}$ in both places results in a question whose answer is not known: an equivalent form of the question is, is there an algorithm for deciding whether a variety over $\mathbb{Q}$ has a rational point? Also, if one replaces $\mathbb{Z}$ by the ring of integers of a number field k, the existence of an algorithm is known only for special k. I will present a survey of some recent progress on these problems.