

# NUMBER THEORY AT THE UNIVERSITY OF ILLINOIS

## Introduction

The Mathematics Department of the University of Illinois at Urbana–Champaign has long been known for the strength of its program in number theory. The department has a large and distinguished faculty noted for their work in this area, and the graduate program in number theory attracts students from throughout the world. At present over twenty students are writing dissertations in number theory. Each semester upper level graduate courses are offered in a variety of topics in analytic, algebraic, combinatorial, and elementary number theory. At least three regularly scheduled seminars are held each week, with lectures being given by faculty, graduate students, and visiting scholars. The lectures may be elementary introductions, surveys, or expositions of current research. Almost every year, our number theorists hold one or two conferences. More details are provided below for each facet of our program.

We begin with a list of the faculty in number theory with brief descriptions of their main interests. At the end of this document, a more detailed account of the research interests of each faculty member is given.

### Faculty in Number Theory

**Scott Ahlgren**, modular forms and connections to number theory, partitions, character sums, Diophantine equations.

**Paul T. Bateman (Emeritus)**, sums of squares, prime number theory.

**Bruce Berndt**, Ramanujan’s notebooks, elliptic functions, theta-functions,  $q$ -series, continued fractions, character sums, classical analysis.

**Florin Boca**, diophantine approximation, spacing statistics.

**Harold G. Diamond (Emeritus)**, prime number theory, sieves, connections with analysis.

**Iwan Duursma**, cryptography, algebraic coding theory, algebraic curves.

**Kevin Ford**, Arithmetic functions, Waring’s problem and variants, Weyl sums, comparative prime number theory, sieve theory, Riemann zeta function.

**Heini Halberstam (Emeritus)**, sieves, arithmetic functions.

**Adolf J. Hildebrand**, analytic and probabilistic number theory, asymptotic analysis.

**Rinat Kedem**, Integrable models in statistical mechanics and conformal field theory; quantum groups and representation theory of affine algebras.

**Leon McCulloh (Emeritus)** Algebraic Number Theory, relative Galois module structure of rings of integers, class groups of integral group rings, Stickelberger relations.

**Walter Philipp (Emeritus; Statistics)**, metric problems in Diophantine approximation, uniform distribution mod 1, continued fractions and other algorithms, probabilistic number theory.

**Bruce Reznick**, sums of squares of polynomials, combinatorial number theory.

**Ken Stolarsky**, diophantine approximation, special functions, geometry of zeros of polynomials.

**Stephen Ullom**, Galois modules, class groups.

**Alexandru Zaharescu**, Analytic and algebraic number theory.

### Courses in Number Theory

Each year, the following one semester courses are offered:

Math 353, Introduction to the Theory of Numbers (upper undergraduate level),

Math 405, Algebraic Number Theory (beginning graduate course),

Math 453, Analytic Number Theory (beginning graduate course).

At least four topics course are also offered each year, with student input helping to determine the choice of the topics courses. In recent years enrollments in these courses have been excellent, typically ranging from 10 to 23. We list below some of the topics courses taught in the past six years:

Algebraic: Class field theory, Advanced topics in the arithmetic of elliptic curves, Elliptic curves by computer, Galois properties of points of finite order on elliptic curves, Galois module theory, Fermat's last theorem, Galois representations, Arithmetic of elliptic curves, Algebraic number theory and computer algebra systems.

Analytic: Methods of classical analysis, Asymptotic methods in analysis, Circle method, Riemann zeta-function, Multiplicative number theory, Analytic and probabilistic number theory, Special functions, Continued fractions, Diophantine approximation, Ramanujan's lost notebook, Elliptic functions with applications to number theory, Modular forms with applications to number theory, Theory of partitions, Transcendental number theory, Gauss and Jacobi sums, Combinatorial number theory, Diophantine problems, Uniform distribution, and Distribution of sequences in number theory.

### Seminars

For over 30 years, each Tuesday and Thursday afternoon, at 1 p.m., number theory seminars have been held. Mainly, these have tended to focus on the analytic and elementary sides of number theory, but talks may be given on any topic relevant to number theory. At 2 p.m. each Thursday, a seminar in algebraic number theory meets. From time to time, special seminars on particular topics in number theory are organized. Talks are intended to be accessible to graduate students, and the atmosphere is informal and supportive with questions and discussion encouraged. Attendance ranges from about 15 to over 25. Students present many of the lectures, with the remainder being given by faculty and visitors.

### Graduate Students

Currently, approximately twenty graduate students at the University of Illinois are writing Ph.D. dissertations in a variety of areas of analytic, algebraic, combinatorial, and elementary number theory. In fact, in recent years about one-third of all advanced graduate students in mathematics have chosen a thesis advisor in number theory. Our recent and current students come from many countries, including the United States, Canada, Korea, and Singapore, Many of these were acquainted with our program in number theory as undergraduates and chose to do their graduate work at Illinois because of our reputation. Several of our current graduate students have written papers, and a list of these papers can be found on our department's web-site. Moreover, during the past five years, over 20 papers were published by

our graduate students in number theory while they were still students in residence here. Some of our recent graduates in number theory have received prestigious postdoctoral awards. For example, Kevin O'Bryant received an NSF postdoctoral fellowship which he is using at the University of California at San Diego. William Galway completed a two year postdoctoral appointment at Simon Fraser University in 2003. Jimmie McLaughlin has a three year postdoctoral fellowship at Trinity College. Yifan Yang received a postdoctoral fellowship at the Institute for Advanced Study Jaebum Sohn and Soon-Yi Kang recently completed postdoctoral positions at Ohio State University. Hamza Yesilyurt received the prestigious John Thompson Assistant Professorship, a three year postdoc position at the University of Florida. Heekyoung Hahn has a postdoctoral position at the University of Rochester. We are fortunate to have many gifted students in number theory, and we feel that we teach them in a caring atmosphere.

### **The Bateman Prize in Number Theory**

Each year the number theory faculty chooses a graduate student in number theory for the Bateman Prize in Number Theory. This honor is given to the student whom the faculty feels has accomplished the most outstanding research during the previous year. The award winners, their advisors, and present positions are:

1. Youn-Seo Choi (Berndt), Korea University, Seoul, Korea.
2. Yifan Yang (Hildebrand), National Chiao Tung University, Taiwan.
3. Mark Bauer (Boston), postdoc at University of Waterloo.
4. Jimmie McLaughlin (Bowman), postdoc at Trinity College, CT.  
Kevin O'Bryant (Stolarsky), postdoc at UCSD.

5. Song Heng Chan (Berndt), graduate student at UIUC.
6. Hamza Yesilyurt (Berndt), Thompson Assistant Professorship at the University of Florida.

### **The Bateman Fellowship in Number Theory**

Each year the number theory faculty chooses a current student in number theory for the Bateman Fellowship in Number Theory. This is a one-semester fellowship given to an outstanding current graduate student in number theory. The recipients of this Fellowship and their advisors have been:

1. Friedrich Littmann (Diamond).
2. Michael Bush (Boston).
3. Song Heng Chan (Berndt).

### **Visitors**

In most years we have at least one visitor in number theory for a semester or full academic year. Recent visitors have included Emre Alkan, Matthew Boylan, Vishwa Dumir, and Ae Ja Yee,

### **Illinois Number Theory Conferences**

Normally, we host conferences in number theory each spring. For most of the past two decades, Illinois Analytic Number Theory meetings have been held over a two day period at the University of Illinois. On four occasions, the analytic meetings were expanded to international conferences of several days. In 1987, a five day meeting commemorating the 100th anniversary of the birth of Ramanujan was held at the University of Illinois. In 1989, we celebrated Paul Bateman's

70th birthday and retirement with a three day meeting at the nearby Conference Center at Allerton Park. In 1995, we returned to Allerton Park to celebrate the 70th birthday and impending retirement of Heini Halberstam. In May, 2000, 276 number theorists attended the Millennial Conference in Number Theory, organized by the number theorists at Illinois. The Illinois Number Theory meeting for 2002 helped celebrate Harold Diamond's retirement. Graduate students are encouraged to present papers at our conferences.

**Faculty in Number Theory at the University of Illinois  
Their Research Interests and Activities**

**Scott Ahlgren**

Ahlgren completed his Ph.D. in 1996 in the field of Diophantine equations under the direction of Wolfgang Schmidt. His early work was on the topic of polynomial-exponential equations. During a postdoctoral appointment at Penn State University his interests broadened, and he began to work on questions in other areas of number theory. Much of his recent research has focused on applications of the theory of modular forms to problems in number theory. For example, one of his major interests has been the study of the arithmetic properties of the usual partition function through the use of modular forms. Another recent focus of his work has been to study connections between modular forms, character sums, and varieties over finite fields. Ahlgren has authored or co-authored seventeen research papers in various areas of number theory.

**Paul T. Bateman (Emeritus)**

Bateman received his Ph.D. in 1946 at the University of Pennsylvania under the supervision of Hans Rademacher. After two-year stints at both Yale University and the Institute for Advanced Study, he joined the number theory group in the University of Illinois Mathematics Department in 1950. He has been at Illinois since then, aside from year-long visits to the Institute for Advanced Study, the University of Pennsylvania, the City University of New York, and the University of Michigan.

Bateman has supervised 20 doctoral dissertations in number theory, one at the University of Colorado and 19 at the University of Illinois. His research has covered a wide range of topics, including sums of squares, the distribution of prime numbers, Beurling's generalized prime numbers, modular forms, geometrical extrema, the coefficients of the cyclotomic polynomials, and arithmetical functions. He has written joint papers with more than twenty different co-authors.

Several members of the current Illinois faculty were appointed during Bateman's fifteen years as Department Head, namely Professors Berndt, Diamond, Janusz, Philipp, Reznick, Stolarsky, and Ullom.

**Bruce Berndt**

Having received his Ph.D. in 1966 at the University of Wisconsin, Berndt is an analytic number theorist with strong interests in several related areas of classical analysis. His primary interests are in special functions, elliptic functions, theta functions,  $q$ -series, the theory of partitions, continued fractions, Eisenstein series, Dirichlet series, and character sums. Since early 1974, almost all of his research has been devoted to proving the claims left without proofs in three notebooks and a "lost notebook" by India's greatest mathematician, Srinivasa Ramanujan, upon his death in 1920. The three notebooks contain approximately 3300 results. The project of

finding proofs for these claims took over twenty years to accomplish, and an account of this work can be found in his books, *Ramanujan's Notebooks, Parts I-V*, published by Springer-Verlag in the years 1985, 1989, 1991, 1994, and 1998. Also during this time, he and Robert A. Rankin wrote *Ramanujan: Letters and Commentary* and *Ramanujan: Essays and Surveys*, both published jointly by the American and London Mathematical Societies. His research in this direction continues, as he and George Andrews plan to publish volumes on Ramanujan's "lost" notebook, analogous to those published on the ordinary notebooks. The lost notebook arises from the last year of Ramanujan's life and contains approximately 650 assertions without proofs. Twentyone students have completed doctoral theses under Berndt's direction, and currently, five Ph.D. students are writing their dissertations under his direction. Most are focusing on material in the lost notebook or on research inspired by Ramanujan. Berndt also has strong interests in other areas of classical analytic number theory, in particular, Dirichlet series, arithmetic functions, and character sums. In 1998, he, Ronald J. Evans, and Kenneth S. Williams published the monograph, *Gauss and Jacobi Sums*.

#### **Florin P. Boca**

Boca completed his undergraduate studies at the University of Bucharest (diploma 1986) and received his Ph.D. at UCLA (1993). He held positions at the Institute of Mathematics of the Romanian Academy (researcher since 1988) and University of Toronto (postdoctoral fellow 1993-1995), and was a EPSRC advanced research fellow in the UK between 1995-1997 (University of Wales Swansea) and 1998-2001 (Cardiff University).

His research, primarily concerned with a series of topics in Operator Algebras (both  $C^*$  and von Neumann algebras), has connected at first with Number Theory in his work on the structure of non-commutative tori and some of their subalgebras. He started working with Alexandru Zaharescu in 1996 on the classification of the Araki-Woods factors associated with adelic products of  $ax+b$  groups of  $p$ -adic fields. This collaboration has broadened since 1999, with progress on several problems from topics as: pair correlation for fractional parts of polynomials, spacing statistics of Farey fractions (co-author C. Cobeli), and asymptotic estimates on the distribution of the free path length in the model of the periodic Lorentz gas (co-author R. N. Gologan).

#### **Harold G. Diamond (Emeritus)**

Diamond received his Ph.D. in 1965 from Stanford University and has been at Illinois since 1967, with visiting appointments at several American and European universities. He became an emeritus faculty member in 2002. His main area of work is multiplicative number theory, particularly elementary proofs of the prime number theorem, the theory of Beurling generalized numbers, and sieve theory. He is coauthor of a Carus monograph on algebraic number theory and a textbook on analytic number theory. Other interests include harmonic analysis and tauberian theorems, numerical computation, and mathematical problems.

Questions about the distribution of prime numbers are at the center of many of Diamond's investigations. The famous Prime Number Theorem (PNT) asserts that  $\pi(x)$ , the number of primes in the interval  $[1, x]$  is asymptotic to  $x/\log x$ . One of Diamond's results is an estimate of the error term in the PNT achieved by 'elementary' methods. Beurling theory is concerned with properties of a collection

of real numbers which has a multiplicative structure but not necessarily an additive one. The central questions concern the relations between Beurling ‘primes’ and ‘integers’ – for example, an analogue of the PNT is known to hold if the ‘integers’ are reasonably well distributed.

The object of sieve theory is to estimate the number of elements that remain in a set if those satisfying certain congruence conditions are removed. Along with Prof. Halberstam and the late H.–E. Richert, Diamond has worked out an improved general sieve in a project of several years’ duration. Interesting questions remain to be studied.

Eleven students completed Ph.D.’s under Diamond’s direction, all employed at universities or scientific organizations.

#### Iwan Duursma

Duursma received Master’s degrees in Aerospace Engineering at Delft University and in Mathematics at University of Amsterdam. He received his Ph.D. in 1993 under the direction of Jack van Lint and Ruud Pellikaan. Part of his thesis work was the formulation and proof of the Feng-Rao algorithm for the decoding of a general geometric Goppa code. Among his current interests in coding theory are: Properties of codes over rings, in particular self-dual codes over  $p$ -adic rings; Weight distributions for asymptotically good codes, relations with zeta functions. In cryptography his interests include most of the algebraically formulated protocols. And in particular those that use number theory (such as RSA) or elliptic curves (such as elliptic curve digital signature schemes). Interests extend to other classes of curves and their Jacobians.

#### Kevin Ford

Kevin Ford has degrees from California State University, Chico (B.S., 1990) and UIUC (Ph.D., 1994). He has held positions at the Institute For Advanced Study, the University of Texas and the University of South Carolina before returning to UIUC in the Fall of 2001.

His research interests cover a variety of topics in elementary, analytic and combinatorial number theory. He has worked on Waring-type problems concerning the representing an integer as a sum of special integers such as perfect  $k$ th powers ( $k \geq 2$ ). These are attacked with the *circle method* using bounds for Weyl sums. Ford also investigates the distribution of values that an arithmetic function takes and also how many times a value is taken. In particular, he proved two 40-year old conjectures of Sierpiński (one jointly with Sergei Konyagin) concerning the sum of divisors function and Euler’s totient function. He works on related problems concerning the the distribution of divisors of integers and prime factors of integers. Important in many of these investigations are *sieve methods*, which are techniques for approximating the number of integers which are free of small prime factors (e.g. primes) lying in “well-distributed” integer sequences. In addition to applying them to other problems, Ford also works on theoretical limitations of sieve methods. Ford also studies properties of the zeros of the Riemann zeta function and other  $L$ -functions, using tools such as Weyl sums, explicit formulas, and properties of trigonometric polynomials. One application of the theory of  $L$ -function is to comparative prime number theory, where one compares the number of primes less than  $x$  lying in two or more arithmetic progressions. Analyzing prime counts in three or more progressions is much more difficult than for two progressions, and this is the focus of ongoing research.

### Heini Halberstam (Emeritus)

Halberstam gained his Ph.D. in 1952 at University College, London University, under the supervision of T. Estermann, on topics in analytic number theory. The contents of the thesis were published in several papers on Waring's problem for mixed powers and on some convolution formulas for certain multiplicative divisor functions proposed by Ingham. From 1949 through 1980 he held various academic appointments in the UK and Ireland; from 1964 to 1980 he was Professor of Mathematics at Nottingham University, being at various times Chair of the Mathematics Department and Dean of the Faculty of Pure Science. During these years he collaborated with K. F. Roth (Imperial College, London) on *Sequences*, a research monograph published by Oxford in 1966 and republished by Springer in 1983, dealing with general properties of integer sequences. Also during this time he wrote *Sieve Methods*, published by Academic Press in 1975, and coauthored with H.-E. Richert (Ulm University, Germany). During this time, he wrote numerous papers on sieves, mostly with Richert, but also with H. Davenport and others on the large sieve. Earlier, in the '50s and '60s, he had worked on gap theorems for  $k$ -free numbers, on the distribution of additive arithmetic functions, and on perfect difference sets.

In 1980 Halberstam moved to the University of Illinois as Professor and (until 1988) as Head of the Mathematics Department. There he continued his work on sieves with Richert but was soon joined by H. Diamond in this enterprise, and together these three developed in a long series of papers a theory for higher dimensional sieves, with many applications. This work has had an interesting overlap with numerical analysis and control theory (specifically, boundary value problems for differential-delay equations). He is emeritus since retirement in 1996, but continues to work on sieves and, more recently, on mean values of multiplicative arithmetic functions.

Halberstam has a long-standing interest in mathematics education. He was a founding member of the Shell Centre for Mathematics Education in Nottingham, and, from 1978 to 1982, member-at-large of the International Commission on Mathematics Education. He has published several essays on this subject.

He has been at various times editor or co-editor of the mathematical works of William Rowan Hamilton (vol. 3), H. Davenport (vol. 4), J. E. Littlewood (vol. 2), and L. K. Hua (a selecta volume); and of three conference proceedings volumes.

Over the years, Halberstam has held research grants from the U.S. Army, NATO, and from the NSF. He has directed the theses of some twenty students.

### A. J. Hildebrand

Hildebrand earned a PhD in 1983 from the University of Freiburg, Germany, and a Doctorat d'Etat in 1984 from the University of Paris-Sud, Orsay, France, and spent a year at the Institute for Advanced Study in Princeton before joining the UIUC faculty in 1986.

Trained as a number theorist, he is also interested in problems in analysis, probability theory, and combinatorics, and, in particular, in problems that lie at the interface of these areas with number theory. Most of his research falls into the areas of analytic number theory, which investigates problems of number theory by methods of analysis, and probabilistic number theory, which studies number theoretic problems of a statistical nature.

Hildebrand has taught special topics courses on asymptotic methods of analysis,

exponential sums, combinatorial number theory, and probabilistic number theory, and has supervised six PhD students.

At the undergraduate level, Hildebrand has a long-standing interest in and involvement with the local mathematical contest scene. For most of the past fifteen years, he and Harold Diamond have served as local coordinators of the William Lowell Putnam Competition and coaches of the UIUC Putnam team, and have organized training sessions, practice contests, and related activities.

### **Rinat Kedem**

Kedem received her Ph.D. in 1993 from the State University of New York at Stony Brook, where she worked in the area of exactly solvable models in statistical mechanics, studying the relationship between critical statistical mechanical systems and conformal field theories, via the character theory of infinite dimensional Lie algebras, and generalizations of the Rogers-Ramanujan identities. She later worked for two years at RIMS in Kyoto University, studying integrable models via their description as representations of quantum affine algebras. She later held positions at the University of Melbourne, University of California, Berkeley, the University of Massachusetts in Amherst and MSRI. She is currently interested in integrable models using the algebraic approach; representations of quantum affine algebras; and combinatorial representation theory of vertex operator algebras and conformal field theories.

### **Leon McCulloh (Emeritus)**

McCulloh received his PhD in 1959 from the Ohio State University under H.B.Mann, and he came to the University of Illinois in 1961. He held visiting appointments at Indiana University and the University of Hawaii in 1963 and 1967, respectively, and a short term appointment at the University of Bordeaux in 1983. He has also visited King's College London, the University of Regensburg, and Cambridge University on sabbatical leaves. He has supervised nine PhD students. His research, starting with his thesis on integral bases in relative Kummer extensions of number fields, has been a logical progression from that beginning. It has centered on relative integral and normal integral bases, and the related notions of realizable Steinitz and Galois module classes. These topics were the basis for the thesis topics of four of his students. He developed a generalization of the notion of Stickelberger relations to class groups of integral group rings and found connections with realizable Galois module classes and with class number formulas for integral group rings. (He owes a heavy debt to Steve Ullom for pointing out the connection between normal integral bases and Stickelberger relations in the work of Hilbert.) In 1987 he published a characterization "in Stickelberger terms" of the realizable Galois module classes of (tame) abelian extensions, in particular showing that they form a subgroup of the classgroup. He has since partially generalized these results to nonabelian extensions but a full generalization is not yet in sight. There are many related open problems and conjectures which he feels provide an important and fruitful area for research.

### **Walter Philipp (Emeritus)**

Philipp's main interests in number theory include metric problems in Diophantine approximation, uniform distribution mod 1, continued fractions and other algorithms, and probabilistic number theory.

Among the main topics in Philipp's research are limit theorems in probability for weakly dependent random variables and their application to problems in number

theory. He has published two volumes in the AMS Memoir series (vols. 114 and 161) dealing with questions in this domain. More recently, in collaboration with I. Berkes, he has obtained results on the discrepancy of sequences  $\{n_k x\}$  for certain sequences  $\{n_k\}$  of integers. In particular, he was able to prove a 20 year old conjecture of Erdős and Gaal, another one of Roger Baker and to disprove a more than 30 year old conjecture of Erdős on this subject.

His work in probability theory deals with the strong approximation of sums of Banach space valued random variables, Hilbert space valued martingales, empirical processes and U-statistics.

### Bruce Reznick

Bruce Reznick's degrees are from Caltech (BS, 1973) and Stanford (Ph.D., 1976). He has been on the faculty of the University of Illinois since 1979. He was a Sloan Foundation fellow from 1983–1986, and received the Prokasy Award for Excellence in Undergraduate Teaching from his College in 1997.

His main research interests originate from questions about the structure of polynomials in several variables and their representations as sums of powers of polynomials and then spread out. The subjects ultimately range from concrete realizations of Hilbert's 17th problem for positive definite forms to Hilbert identities; from spherical designs and quadrature formulas to the linear algebra of polynomials and the  $L_\infty$  norm of the Laplacian on spaces of forms of fixed degree; from polynomial solutions of Diophantine equations to concrete versions of theorems in abstract real algebraic geometry; from questions about the lattice point structure of polytopes with lattice point vertices to structure theorems for recursively defined sequences. So far, these have encompassed questions in number theory, algebra, analysis and combinatorics.

### Kenneth B. Stolarsky

Stolarsky did his undergraduate studies at Caltech and received his Ph.D. at the University of Wisconsin (Madison). His research interests include number theory, geometry, classical analysis, and combinatorics, and he has supervised four Ph.D. theses in these areas. Stolarsky is particularly interested in Diophantine approximation and has taught many graduate courses at UIUC in Diophantine approximation and the related areas of transcendence theory and the geometry of numbers. His undergraduate teaching has centered on introductory differential equations, and he has also done much editorial work for the problems section of the *American Mathematical Monthly*.

### Stephen Ullom

Ullom received his Ph.D. in 1968 from the University of Maryland; his thesis advisor was Sigeokatu Kuroda. He spent his NSF Postdoctoral Fellowship (1968–69) at the University of Karlsruhe and King's College, University of London, his mentors being H. W. Leopoldt and A. Fröhlich respectively. In 1969–1970 he was a visiting member of the Institute for Advanced Study. He came to the University of Illinois in 1970 as an Assistant Professor and was promoted to Professor in 1978. Ullom has spent sabbatical leaves at King's College London and Cambridge University (Bye Fellow of Robinson College) with a shorter visit to the University of Arizona. He has supervised three Ph.D. students.

In his thesis Ullom studied the Galois module structure of ideals in a local or global field which are invariant under the Galois group. He found the first example

where the ring of integers is not projective over the integral group ring, but some ideals are projective. He generalized Hilbert's theorem on the connection between Galois module structure and Stickelberger elements to proper ideals. Frequently in collaboration with Irv Reiner, Ullom studied class groups of integral group rings, particularly on their arithmetic properties. He proved a conjecture of Kervaire and Murthy on the class groups of cyclic  $p$ -groups by using Iwasawa theory applied to group rings. His work on Swan modules, a particularly simple form of projective module, showed most noncyclic groups  $G$  have projective nonfree modules over the group ring  $ZG$ . Tate cited Ullom's 1977 survey article on class groups in his book on Stark's conjecture. Ullom's student, Steve Watt, extended Fröhlich's results on Galois groups of  $p$ -extensions with restricted ramification over the rationals to the case of imaginary quadratic base field. Ullom and Watt used this result to give a criterion for when an abelian  $p$ -extension  $L$  of an imaginary quadratic field  $K$  has class number prime to  $p$  (assuming the genus number for  $L/K$  is prime to  $p$ ).

Ullom wrote a paper with N. Boston on Galois deformations associated to elliptic curves with complex multiplication. An important consequence is that for the Fermat curve there are infinitely many primes  $p$  such that the deformation ring is not simply a ring of formal power series in several variables.

Marcin Mazur and Ullom investigated the Galois module structure of units modulo torsion in certain real abelian fields of two power degree. There are many fascinating connections here between Galois module structure and classical topics such as genus field, central class field, and sign of the fundamental unit in a quadratic field. At present Ullom is also investigating some problems on the structure of Galois groups of extensions of number fields with restricted ramification.

### Alexandru Zaharescu

Zaharescu received his Ph.D. in 1995 at Princeton University under the supervision of Peter Sarnak. Since then he has held positions at Massachusetts Institute of Technology, McGill University and the Institute for Advanced Study. His early work centered on valuation theory and local fields (especially Galois theory and class field theory). After he started working with Sarnak, his research interests shifted more to the analytic side of number theory, the main object of his later investigations being  $L$ -functions and their applications. In recent years his interests broadened, and he is currently working on a variety of topics in number theory, such as: primitive roots, character sums and exponential sums, integer points on or near algebraic curves, Siegel zeroes, distribution mod 1, metric invariants over local fields, Farey sequences,  $p$ -adic rigid analytic geometry and distribution of zeroes of  $L$ -functions.