

# PARTITIONING THE EDGES OF A PLANAR GRAPH INTO THOSE OF A FOREST AND A GRAPH OF LIMITED VERTEX DEGREE

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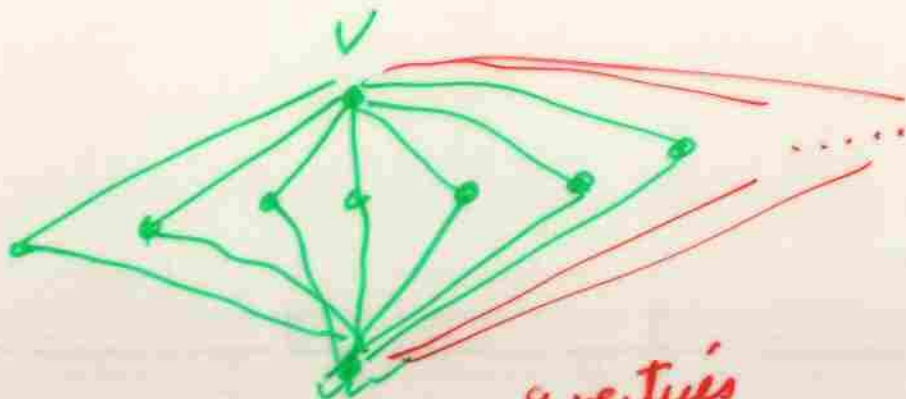
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JGT 41 307-3

My aim -

to acquaint you with some  
nice open problems,  
will prove the result to give ~~you~~ <sup>of problem</sup>  
work by me with help from several  
students, including

David Vincent  
Pavel Pylauski  
Jonathan Comstock  
Bridget Turner

There is no general limit <sup>fixed</sup> on the maximum vertex degree of edges that must be removed to make a planar graph into a forest:



This graph has 9 vertices

7 faces.  
in general  $N$  vertices  $+ N - 2$  faces.

You have to remove 6 or in general  $N - 3$  edges to get a tree and one of  $v$  and  $w$  then has to get degree at least  $\frac{N-3}{2}$ .

However if we restrict the girth <sup>to 5</sup> to be  $\geq 5$ , there is a bound,

Easy result:

If the girth of  $G$  is 11 or more,  
 The edges of  $G$  can be split  
 into those forming a forest  
 and those forming a partial matching.

H-H-L -S-W-Z proved

	Girth		7	8	9	10	11
bound on Max Degree among $G$ edges - Forest edges	$\leq 4$	$\leq 4$	$\leq 2$	$\leq 2$	$\leq 2$	$\leq 2$	$\leq 1$
New Results	<u>3 or 4</u> open	2	2	<u>1 or 2</u> open	<u>1 or 2</u> open		1

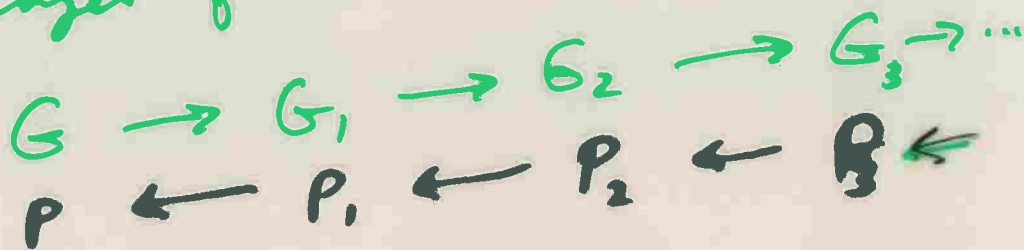
will prove this one

# Approach -

Show that there can be no minimum counterexample to the claim.

Here this provides a constructive algorithm for finding a partition as desired -

You find a reduction from  $G$  to a "smaller" graph, and continue reducing until the partition is obvious - then undo the reductions so as to partition the edges of  $G$ .



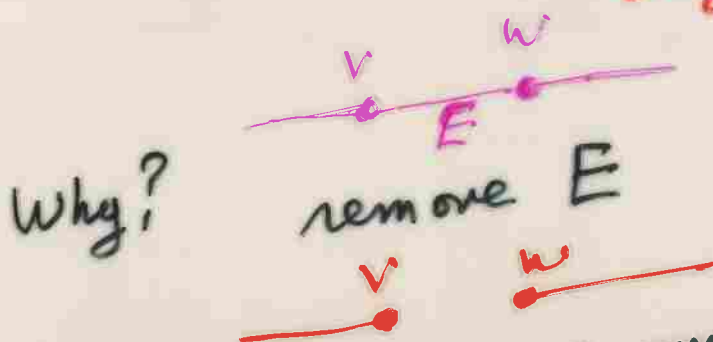
What Reductions?

Suppose you want to prove that the edges of  $G$  can be partitioned into those of a forest  $F$  and those of a graph  $D$  of maximum vertex degree  $k$ .

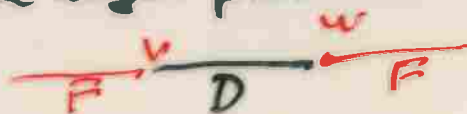
For example let  $k=1$

Then  $G$  cannot be a minimum counterexample to the existence of a such partition

if it possesses an edge  $E$  both of whose vertices have degree 2



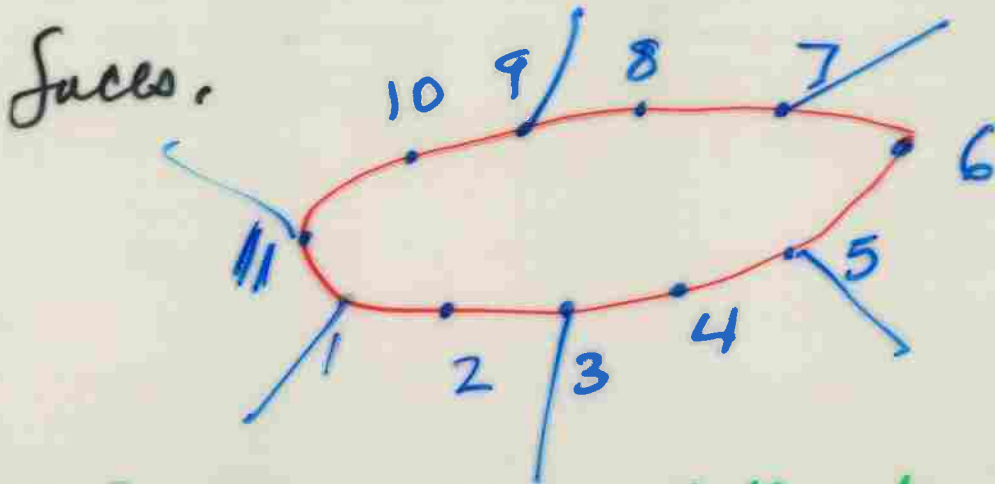
Now, if  $G$  were a minimum counterexample (in any sense) <sup>reasonable</sup>  
 Its <sup>remaining</sup> edges can be partitioned, and without loss of generality we can put the edges pictured in the forest.



This is a very simple reduction but it already tells us that the edges of a planar graph of girth 11 can be partitioned into a forest and a matching.

Why?

If  $G$  has girth 11 and has no adjacent vertices of degree 2, every face must be adjacent to at least 6 other



So every vertex of the dual of  $G$  has degree at least 6.

But for most edges a planar graph on  $F$  vertices can have  $\leq 3F - 6$

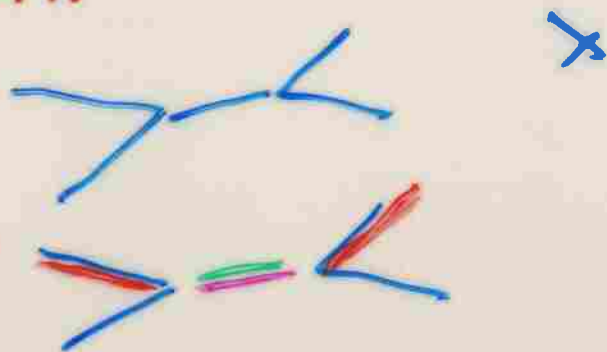
Our plan is to try to mimic this argument.

How:  
First generalize the given reduction

If To prove that  $G$  cannot be a minimum counterexample to a partition into a forest and a graph  $D$  of maximum vertex degree  $k$

Note that  $G$  cannot contain 2 adjacent vertices both of degree  $\leq k+1$ .

eg  $k=2$

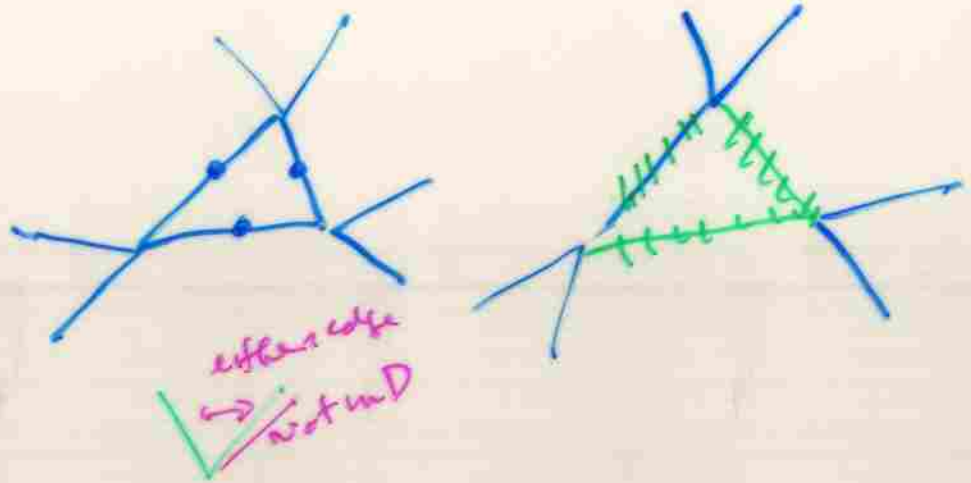



↑ in forest

↑ in  $D$

Further generalization: -

$G$  cannot be a minimum Counterexample  
 if it contains a cycle with  
 Every second vertex of degree 2  
 the rest of its vertices of degree  $\leq k+2$




  
 remove the triangle

