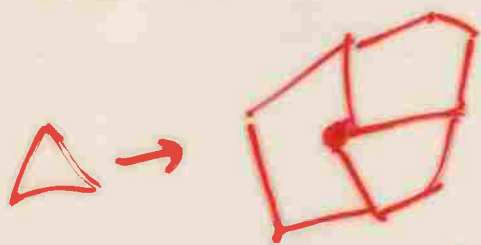


We want to emulate
the simple growth argument,

If we look at the dual
graph to G , (ignoring vertices
of G of degree 2) we find:

A \triangle face in G

becomes a vertex of degree 3
the faces around it in the dual
are 4+ sided:



(A Non square
face \rightarrow
(for example))

We want to use the condition
that there must be vertices
in this dual that have "effective
degree" less than 6

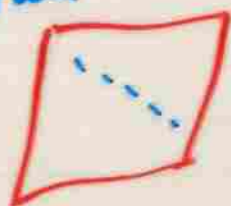
By "effective degree"

we mean:

Degree assigned to that dual vertex (originally a face) after filling in the ^{dual} faces here to make a triangulation,

The sum of vertex degrees = $6V - 12$ for a triangulation

4 face



5 face



6 face



And we can allocate the extra vertex degrees created by adding edges to faces ANYWAY we want among dual vertices,

We want to allocate ^{face degrees} so that every dual vertex gets effective degree ≥ 6 if G is minimally not partitionable which would mean that G is not planar

Every original non-degree 2 vertex of G has degree at least 4 in dual - this is at least a 4 face



We can add 1 edge inside it adding 2 to the sum of vertex degrees. We allocate $\frac{1}{2}$ to each dual-vertex

This means that an original non-triangle face f_i in the dual looks like



and has actual (dual) degree ≥ 4 and allocated degree ≥ 2 for a total of at least 6.

A triangle dualizes to



and gets

allocated $3 + \frac{3}{2}$ or $4\frac{1}{2}$

NOT ENOUGH!

A degree 5 vertex

X also a 5 sided face
in the dual

And we can add 2 edges
and so vertex degree 4
after allocating $\frac{1}{2}$ to each dual
vertex, we have $1\frac{1}{2}$ degree left
over, which we can allocate
to triangles.

If a 5-face meets only 1
triangle, its extra $1\frac{1}{2}$ is allocated to
that triangle which then has effective
degree 6.

A 6-face similarly has
leftover degree $6 - 3 = 3$

which can give up
to 2 incident triangles allocated
degree 6.

A degree 7 dual face
has leftover degree $4\frac{1}{2}$ which can
allocate $1\frac{1}{2}$ to each of 3 triangles
 $8 \rightarrow 6 \rightarrow 4$ triangles, etc.

And so?

If there are no back to back triangles (ie "diamonds")

then the only way a face can have allocated degree < 6

is if it is:

1. A triangle
2. either surrounded by vertices of degree 4 (ie dual faces that are 4-sided) or with a configuration of triangles joined at degree 5 or degree 6 vertices with each degree 6 vertex incident to 2 triangles and each degree 5 vertex incident to 3 triangles.

These are exactly the configurations that we found to be reducible - not possible in a minimum counterexample.

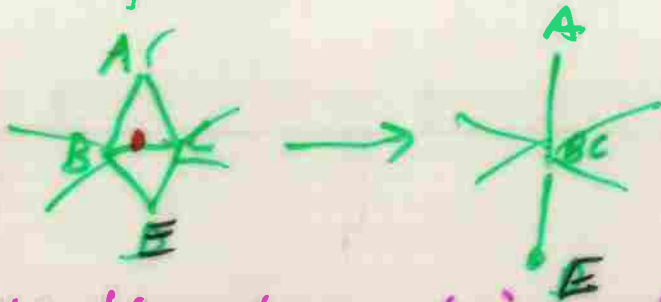
So unless G has a diamond



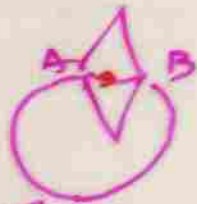
and is planar, it is reducible,

And what about diamonds?

We can reduce a diamond to a path

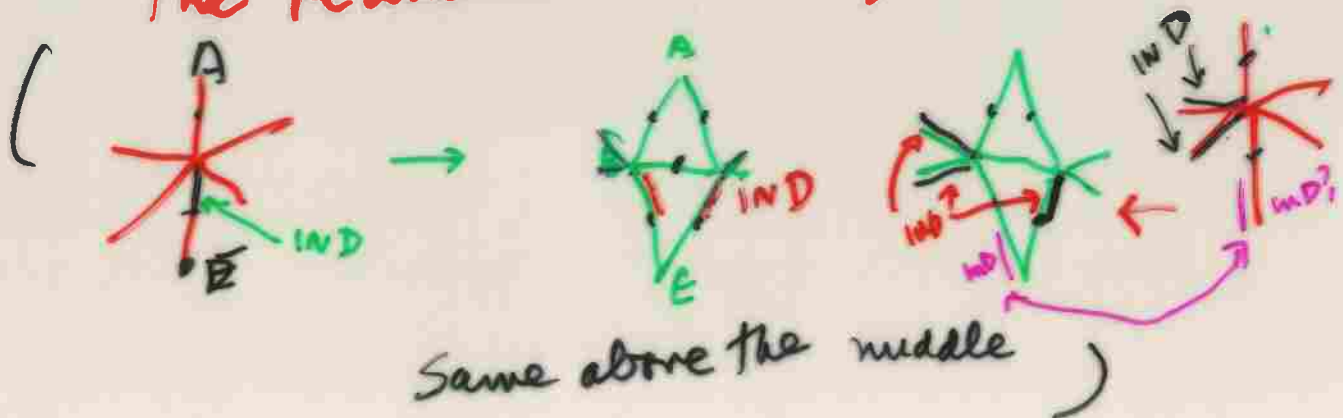


Unless the diamond is set in a ring



path from A to B of length 4 or 5

The reduction is easy



But suppose there are diamonds
in rings?



We can show

1. If G has diamonds
in rings it has a diamond ring
with no diamonds inside its ring.
Call such a ring - a "fitted ring"



(because any diamond
inside either has no ring
or has one that
stays inside the ring -

So these rings nest
and there must be one with
no diamond inside it.)

2. If we throw away all of G
outside a fitted ring - there must
be ^(closed vertices) faces with degree less than 6 into
interior,