New coloring applications of the Regularity Lemma

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Outline

1. Notation and definitions
2. Three-color Ramsey numbers for paths
3. Vertex disjoint monochromatic cycle partition numbers
4. Monochromatic hypergraph cycles
5. The Regularity Lemma – Blow-up Lemma method
6. Main ideas of the proofs

1. Notation and definitions

K_n is the complete graph on n vertices. $K(u,v)$ is the complete bipartite graph between U and V with $|U|=u$, $|V|=v$.

When A, B are disjoint subsets of $V(G)$, we denote by $e(A,B)$ the number of edges of G with one endpoint in A and the other in B. For non-empty A, B

$$d(A,B) = \frac{e(A,B)}{|A|\,|B|}$$

is the density of the graph between A and B.

$\delta(G)$ = minimum degree in G

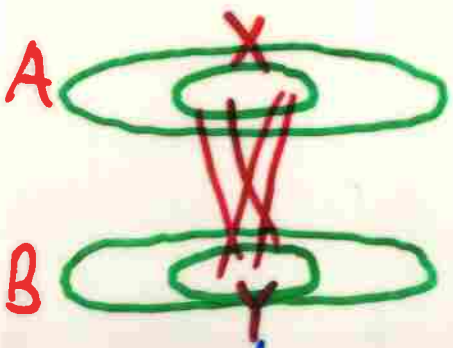
$\Delta(G)$ = maximum " "

The bipartite graph $G(A,B)$ (or simply the pair (A,B)) is called (ε,δ)-regular if

$$X \subset A,\ Y \subset B,\ |X| > \varepsilon|A|,\ |Y| > \varepsilon|B|$$

imply

$$|d(X,Y) - d(A,B)| < \varepsilon$$



$G(A,B)$ is (ε,δ)-super-regular, if it's ε-regular and

$$\deg(a) \geq \delta|B| \quad \forall\ a \in A$$

$$\deg(b) \geq \delta|A| \quad \forall\ b \in B$$