

2. Three-color Ramsey numbers for paths

$R(G_1, G_2, \dots, G_r)$ is the minimum n such that an arbitrary r -edge coloring of K_n contains a copy of G_i in color i for some i .

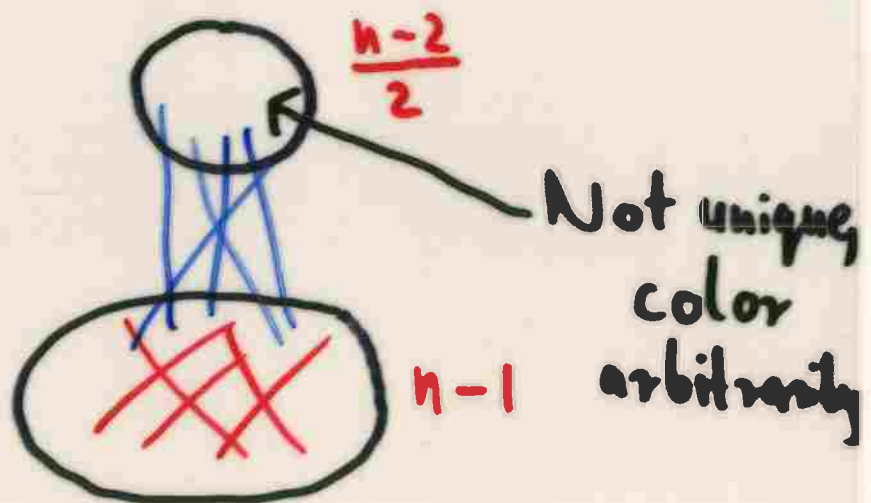
Very little is known about $R(G_1, \dots, G_r)$ in general, there are some results for special graphs.

Gerencsér-Gyárfás 1967

$$R(P_n, P_n) = \left\lfloor \frac{3n-2}{2} \right\rfloor$$

Sharpness:

n even

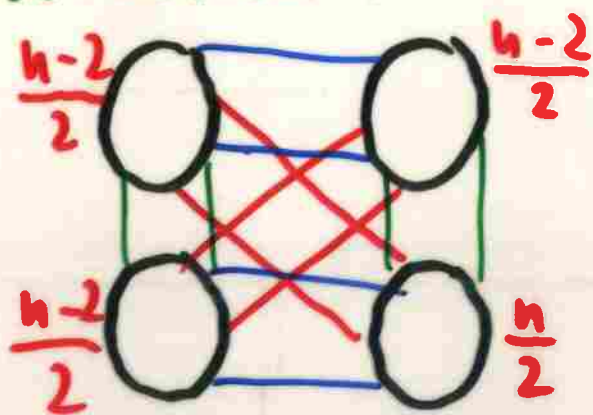


Conjecture ("feeling") of Faudree and Schelp 1975:

$$R(P_n, P_n, P_n) = \begin{cases} 2n-1 & \text{odd } n \\ 2n-2 & \text{even } n \end{cases}$$

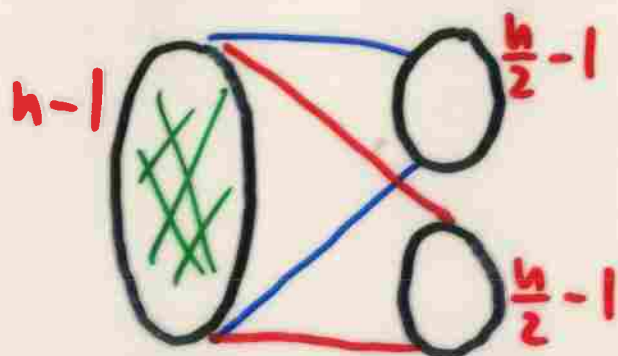
Sharpness: n even

Construction 1:



color inside the classes arbitrarily

Construction 2:



on the right side color arbitrarily

Remark: $R(\overbrace{P_n, P_n, \dots, P_n}^{\gamma}) \leq \gamma n$

follows from density result (Erdős-Gallai)

Łuczak 1999

$$R(C_n, C_n, C_n) = \begin{cases} (4 + o(1))n & \text{odd } n \\ \leq (3 + o(1))n & \text{even } n \end{cases}$$

Ideas for $(2 + o(1))n$ for even n

Kohayakawa, Simonovits, Skokan 2005

$$R(C_n, C_n, C_n) = 4n - 3 \quad \text{odd } n \geq n_0$$

Figaj, Łuczak 2004

$$\begin{aligned} R(C_n, C_n, C_n) &= (2 + o(1))n && \text{even } n \\ R(P_n, P_n, P_n) &= (2 + o(1))n && \forall n \end{aligned}$$

Independently:

Gyárfás, Ruszinkó, G.S., Szemerédi 2004

$$R(P_n, P_n, P_n) = (2 + o(1))n \quad \forall n$$

Finally:

Theorem 1. (Gyárfás, Ruszinkó, G.S., Szemerédi 2005)
There exists an n_0 such that Combinatorica

$$R(P_n, P_n, P_n) = \begin{cases} 2n - 1 & \text{odd } n \geq n_0 \\ 2n - 2 & \text{even } n \geq n_0 \end{cases}$$

3. Vertex disjoint monochromatic cycle partition numbers

$p(r)$ is the minimum number of monochromatic vertex disjoint cycles needed to partition the vertex set of any r -edge colored K_n .
Single vertices and edges are cycles.
It is not obvious that $p(r)$ is well-defined:

Erdős, Gyárfás, Pyber 1991

$$p(r) \leq c r^2 \log r$$

Conjecture (Gyárfás)

$$p(r) = r$$

Łuczak, Rödl, Szemerédi 1998

The conjecture is proved

for $r=2$ and $n \geq n_0$.

Generalizations:

$P_b(r)$: $K(n,n)$ instead of K_n

$P(r,k)$: partition with vertex disjoint connected monochromatic k -regular subgraphs.

$P_b(r,k)$

Haxell 1997

$$P_b(r) \leq c(r \log r)^2$$

G.S., Selkow 2000

$$P(r,k), P_b(r,k) \leq r^{c(r \log r + k)}$$

Now an improvement on the EGYPT result:

Theorem 2. (Gyárfás, Ruszinkó, G.S., Szemerédi; JCTB 2005)

For every $r \geq 2$ there exists $n_0 = n_0(r)$ such that $100 r \log r$ cycles are

enough for $n \geq n_0$.