4. Monochromatic hypergraph cycles

(ongoing research with Gyárfás, Lehel and Schelp)

$K_n^{(r)}$ is the complete r-uniform hypergraph.

There are several natural definitions for a hypergraph cycle.

Loose cycles in $K_n^{(3)}$

C_m is a loose cycle in $K_n^{(3)}$, if it has vertices $\{v_1, \ldots, v_m\}$ and edges $\{v_1v_2v_3, v_3v_4v_5, v_5v_6v_7, \ldots, v_{m-1}v_mv_1\}$ (so m is even)

If $\mathcal{H}$ is a 3-uniform hypergraph:

$$N(x,y) = \{z \mid (x,y,z) \in E(\mathcal{H})\}$$

Minimum degree:

$$\delta(\mathcal{H}) = \min_{x,y} |N(x,y)|$$