



Consider an object of mass m sliding without friction on a mountain range given by the graph of the function $y = f(x)$, where $x = x(t)$ is the horizontal coordinate of the object, and t is the time variable.

1. Find a second-order differential equation for $x(t)$ that describes the motion of the object. Hint: Look at the gravitational force F_g acting on the object. Decompose it into a component F_P that's perpendicular to the mountain range and a component F_T that's tangent to the mountain. Only one of those two components will have an effect on the object. Decompose this one further, into a horizontal component F_H and a vertical component F_V . Which of those two components will affect $x(t)$? Use Newton's second axiom to write down a differential equation for $x(t)$.
2. Recall that for small angles α , we have $\sin \alpha \approx \alpha \approx \tan \alpha$ and $\cos \alpha \approx 1$. Assuming that $f'(x)$ is small, use these approximations to simplify the differential equation for $x(t)$.
3. Let $f(x) = x^2$. Find the general solution of the simplified equation for this choice of $f(x)$, then find the solution that satisfies $x(0) = 0$ and $x'(0) = 1$. What kind of behavior do you expect? Does the solution agree with your expectations?