Math 125 – Exam 1 – Version 1
February 20, 2006

1.
(a) (7pts) Find the points of intersection, if any, of the following planes.

$$\begin{aligned} -3x + 9y + 6z &= 3 \\ 2x - 6y - 4z &= -2 \\ -x + 3y + 2z &= 1 \end{aligned}$$

Solution:

$$\textit{augmented} \qquad \left[\begin{array}{rrr|r} -3 & 9 & 6 & 3 \\ 2 & -6 & -4 & -2 \\ -1 & 3 & 2 & 1 \end{array}\right]$$

$$R_1 \leftrightarrow R_3 \qquad \left[\begin{array}{rrr|r} -1 & 3 & 2 & 1 \\ 2 & -6 & -4 & -2 \\ -3 & 9 & 6 & 3 \end{array}\right]$$

$$\begin{array}{l} R_1' = -R_1 \\ R_2' = R_2 - 2R_1' \\ R_3' = R_3 + 3R_1' \end{array} \qquad \left[\begin{array}{rrr|r} 1 & -3 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Finding the points of intersection is equivalent to finding the solution set of this system of equations. Note that both the y and z variables correspond to columns without leading 1's. Therefore, we define these as parameters in the solution set. Let $y = s$ and $z = t$. The first row of the reduced row echelon form matrix says $x - 3s - 2t = -1$ or $x = 3s + 2t - 1$. Putting this information together, we get

$$(3s + 2t - 1, s, t) \text{ for all } s \text{ and } t.$$

(b) (3pts) Identify the intersection of the planes as the empty set, a point, a line, or a plane. Justify your answer.

Solution: Since the solution set has two parameters, we know that the solution set from part (a) defines the equation of a plane.

2.

(a) (3pts.) Clearly state the theorem used in determining whether or not a system of equations is consistent.

Solution: A linear system is consistent if and only if the row rank of the coefficient matrix equals the row rank of the augmented matrix.

(b) (7pts.) Use the theorem to determine if the following system is consistent.

$$\begin{aligned} x_1 - 2x_4 &= -3 \\ 2x_2 + 2x_3 &= 0 \\ x_3 + 3x_4 &= 1 \\ -2x_1 + 3x_2 + 2x_3 + x_4 &= 5 \end{aligned}$$

Solution:

$$\textit{augmented} \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{array}\right]$$

$$R_4' = R_4 + 2R_1 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{array}\right]$$

$$\begin{array}{l} R_2' = \frac{1}{2}R_2 \\ R_4' = R_4 - 3R_2' \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{array}\right]$$

$$R_4' = R_4 + R_3 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Now that the matrix is in row echelon form, we can determine the ranks. The row rank of the coefficient matrix is 3. Similarly, the row rank of the augmented matrix is 3. By the theorem, since the row ranks are the same, the system is consistent.

3. (1pt each) Answer the following either True or False and justify your answer.

(a) Every elementary row operation is reversible.

Solution: True. If you switch to rows, you can switch them back. If you multiply a row by a non-zero scalar k, you can divide the same row by k. etc.

(b) If a 3×5 coefficient matrix for a system has three leading 1's, then it is consistent.

Solution: True. If the coefficient matrix has three leading 1's, then the rank of the coefficient matrix is 3. Since there are only three rows total in the augmented matrix, the augmented matrix must have rank 3 as well. By theorem, the system is consistent.

(c) An inconsistent system has more than one solution.

Solution: False. The definition of an inconsistent system is that the linear system has NO solution.

(d) In some cases, by using different sequences of row operations, a matrix may be row reduced to more than one matrix in reduced row echelon form.

Solution: False. The reduced row echelon form of a matrix is unique.

(e) Gaussian elimination applies only to augmented matrices for a linear system.

Solution: False. Gaussian elimination is a technique for moving a matrix into row echelon form. This applies to any matrix, not just augmented matrices. In fact, in both Homework 2 and 3 we applied Gaussian elimination to non-augmented matrices.

4.
(a) (4pts.) Define the row rank of a matrix.

Solution: The row rank of a matrix is the number of non-zero rows that a matrix has after it has been moved to row echelon form.

(b) (3pts each) Determine the row rank of the following matrices.

1. $\left[\begin{array}{rrr|r} 1 & -4 & 7 & 2 \\ 0 & 3 & -5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right].$

   Solution: This matrix in row echelon form is $\left[\begin{array}{rrr|r} 1 & -4 & 7 & 2 \\ 0 & 1 & -5/3 & -7/3 \\ 0 & 0 & 0 & 1 \end{array}\right]$. There are 3 non-zero rows, hence the row rank is 3.

2. $\begin{bmatrix} 1 & -4 & 6 \\ 0 & 1 & 5 \\ -2 & 7 & -17 \end{bmatrix}.$

   Solution: This matrix is also not in row echelon form.

$$R3' = R_3 + 2R_1 \quad \begin{bmatrix} 1 & -4 & 6 \\ 0 & 1 & 5 \\ 0 & -1 & -5 \end{bmatrix}$$

$$R3' = R_3 + R_2 \quad \begin{bmatrix} 1 & -4 & 6 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

   This matrix has 2 non-zero rows, hence it has a row rank of 2.

5. (5pts) Consider the following system of linear equations:

$$\begin{aligned} x + y + z &= 1 \\ x + 3z &= -2 + s \\ x - y + rz &= 3 \end{aligned}$$

For what values of r and s is this system of linear equations inconsistent?

Solution: To determine consistency, we must make the augmented matrix that corresponds to this system, move the matrix into row echelon form (or as near as we can get with the variables r and s) and appeal to the theorem on consistency.

$$\textit{augmented} \qquad \left[\begin{array}{rrr|r} 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & -2+s \\ 1 & -1 & r & 3 \end{array}\right]$$

$$\begin{array}{r} R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{array} \qquad \left[\begin{array}{rrr|r} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -3+s \\ 0 & -2 & r-1 & 2 \end{array}\right]$$

$$R_2' = -R_2 \qquad \left[\begin{array}{rrr|r} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 3-s \\ 0 & -2 & r-1 & 2 \end{array}\right]$$

$$R_3' = R_3 + 2R_2 \qquad \left[\begin{array}{rrr|r} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 3-s \\ 0 & 0 & r-5 & 8-2s \end{array}\right]$$

Without knowing more about r, this is as far as we can reduce. By theorem, we know that the row rank of the coefficient matrix must equal the row rank of the augmented matrix for the system to be consistent. Since the question asks for when in the system inconsistent, we know that we want the row rank of the coefficient matrix to be 2 while we want the row rank of the augmented matrix to be 3. The coefficient matrix is of rank 2 when $r - 5 - 0$ or

$$r = 5.$$

When $r = 5$, for inconsistency we require that $8 - 2s$ equal anything but 0. That is, we want $8 - 2s \neq 0$. Solving for s, we see that we need

$$s \neq 4.$$

In summary, when $r = 5$ and $s \neq 4$ the system of equations will be inconsistent.

6. (10pts) A coal company has two mines, a surface mine and a deep mine. It costs \$2500 a day to operate the surface mine and \$6000 dollars a day to operate the deep mine. Each mine produces a medium grade and a medium-hard grade coal, but in different proportions. The surface mine produces 12 tons of medium grade and 6 tons of medium-hard grade coal a day, and the deep mine produces 4 tons of medium grade and 8 tons of hard grade a day. The company has a contract to deliver at least 600 tons of medium grade and 480 tones of medium-hard grade coal within 60 days. Set up **but do not solve** a linear program that will minimize cost.

Solution: The above information can be organized in the following table:

	medium	medium-hard	cost
surface mine	12 tons	6 tons	\$2500
deep mine	4 tons	8 tons	\$6000

(Note: You did not have to table the information for credit.)

Let s = the number of days operating the surface mine and d = the number of days operating the deep mine.

The objective here is to minimize cost. The cost function is $C = 2500s + 6000d$.

The order is to deliver 600 tons of medium grade and 480 tons of medium-hard grade coal. The medium grade inequality is

$$12s + 4d \geq 600.$$

Similarly, the medium-hard grade inequality is

$$6s + 8d \geq 480.$$

Lastly, we know that our variables need to be non-negative. (It doesn't make sense to run a mine a negative number of days.) Due to the 60 day deadline on the order, we get the following constraints on s and d:

$$0 \leq s \leq 60 \text{ and } 0 \leq d \leq 60.$$

The above discussion accurately describes the linear program. One could choose to represent this in its mathematical form:

$$\begin{array}{ll} \text{maximize} & C = 2500s + 6000d \\ \text{subject to} & 12s + 4d \geq 600 \\ & 6s + 8d \geq 480 \\ & 0 \leq s \leq 60 \\ & 0 \leq d \leq 60 \end{array}$$

(Note: You did not have to order the linear program like this for credit.)

7. Consider the linear program:

$$\begin{array}{ll} \text{maximize} & z = -2x + y \\ \text{subject to} & x + y \leq 4 \\ & y \leq 2 \\ & x - y \leq 2 \\ & x \geq 0, y \geq 0 \end{array}$$

(a) (2pts) Construct the initial simplex table for this system.

Solution: Moving the objective function into standard form and adding slack variables to the constraint inequalities, we get:

$$\begin{aligned} z + 2x - y &= 0 \\ x + y + s_1 &= 4 \\ y + s_2 &= 2 \\ x - y + s_3 &= 2 \end{aligned}$$

where s_1, s_2, and s_3 are slack variables.

Constructing the initial simplex table:

$$\left[\begin{array}{c|cc|ccc|c} 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 1 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 & 0 & 1 & 2 \end{array}\right].$$

(b) (2pts) Determine the first pivot point dictated by the simplex algorithm.

The -1 heading the y column is the most negative entry in the top row. Therefore, we look for a pivot point in this column. Recall that we can only pivot on the positive entries.

`pivot candidate`	`quotient`
1	$\frac{4}{1} = 4$
1	$\frac{2}{1} = 2$

Since the 1 in the third row makes the smallest quotient when divided into the entry in the last column, it is the pivot point.

(c) (4pts) Perform the first pivot dictated by the simplex algorithm and determine the basic feasible solution for the resulting table. For full credit, perform each step in the pivot separately. *(Do not just use your calculator. I am testing to see if you know what pivoting is.)*

Solution:

$$I.S.T. \quad \left[\begin{array}{c|cc|ccc|c} 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 1 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$R_1' = R_1 + R_3 \quad \left[\begin{array}{c|cc|ccc|c} 1 & 2 & 0 & 0 & 1 & 0 & 2 \\ \hline 0 & 1 & 1 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$R_2' = R_2 - R_3 \quad \left[\begin{array}{c|cc|ccc|c} 1 & 2 & 0 & 0 & 1 & 0 & 2 \\ \hline 0 & 1 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$R_4' = R_4 + R_3 \quad \left[\begin{array}{c|cc|ccc|c} 1 & 2 & 0 & 0 & 1 & 0 & 2 \\ \hline 0 & 1 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 & 1 & 4 \end{array}\right]$$

The basic feasible solution for this table is $x = 0$, $y = 2$, $s_1 = 2$, $s_2 = 0$, and $s_3 = 4$ or $(0, 2, 2, 0, 4)$.

(d) (2pts) Is the z value determined by simplex table in (c) a maximum? Clearly explain your answer.

Solution: Yes, this table shows the maximum z value. We know this because there are no longer any negative entries in the top row of the simplex table. Hence, this is the final simplex table.

Math 125 – Exam 1 – Version 2
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1. (5pts) Find an equation involving g, h, and k that makes this augmented matrix correspond to a consistent system:

$$\left[\begin{array}{rrr|r} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right].$$

Solution: We need to move this augmented matrix into row echelon form.

$$R_3' = R_3 + 2R_1 \quad \left[\begin{array}{rrr|r} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{array}\right]$$

$$R_3' = R_3 + R_2 \quad \left[\begin{array}{rrr|r} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{array}\right]$$

$$R_2' = \tfrac{1}{3}R_2 \quad \left[\begin{array}{rrr|r} 1 & -4 & 7 & g \\ 0 & 1 & -5/3 & h/3 \\ 0 & 0 & 0 & k+2g+h \end{array}\right]$$

The row rank of the coefficient matrix is 2. By theorem, we know that the row rank of the augmented matrix must be 2 as well if the system is going to be consistent. This requires that

$$k + 2g + h = 0.$$

If g, h, and k satisfy this equation, the linear system will have solutions.

2.
(a) (3pts) Define what it means for a system of linear equations to be consistent.

Solution: A linear system is consistent if it has one or more solutions.

(b) (7pts) Determine if the following system is consistent.

$$\begin{aligned} x_1 + 3x_3 &= 2 \\ x_2 - 3x_4 &= 3 \\ -2x_2 + 3x_3 + 2x_4 &= 1 \\ 3x_1 + 7x_4 &= -5 \end{aligned}$$

Solution:

$$\textit{augmented} \quad \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right]$$

$$R_4' = R_4 - 3R_1 \quad \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right]$$

$$R_3' = R_3 + 2R_1 \quad \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right]$$

$$\begin{array}{l} R_3' = \frac{1}{3}R_3 \\ R_4' = R_4 + 3R_3 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 1 & -4/3 & 7/3 \\ 0 & 0 & 0 & -5 & 10 \end{array}\right]$$

$$R_4' = -\frac{1}{5}R_4 \quad \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 1 & -4/3 & 7/3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

Since the row rank of the coefficient matrix is 4 and equals the row rank of the augmented matrix, the system is consistent.

3. (1pt each) Answer the following either True or False and justify your answer.

(a) Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

Solution: True. This is exactly why we have only the three elementary row operations. These operations correspond to the appropriate operations on equations that do not change the solution of the system of equations.

(b) Gauss-Jordan elimination applies only to augmented matrices for a linear system.

Solution: False. Gauss-Jordan elimination is a method of moving ANY matrix into reduced row echelon form. It is not specific to an augmented matrices for a linear system.

(c) The row echelon form of a matrix is unique.

Solution: False. (This was a homework question.) The reduced row echelon form of a matrix is unique. Every row echelon form of a matrix has the leading 1's in the same positions within the matrix but the entries above the leading 1's can differ.

(d) If a 3×5 coefficient matrix for a system has two leading 1's, then it is inconsistent.

Solution: False. Two leading 1's tell us the row rank of the coefficient matrix is 2 but without knowing the row rank of the augmented matrix, we are unable to determine consistency. If the row rank of the augmented matrix is 2 as well, the system will be consistent.

(e) A homogeneous system is always consistent.

Solution: True. The trivial solution satisfies every homogeneous system.

4.

(a) (4pts) Describe the difference between the row echelon form of a matrix and the reduced row echelon form.

Solution: The row echelon form of a matrix has zeros below every leading 1 in the matrix. The reduced row echelon form of a matrix is a matrix in row echelon form with the additional property that above every leading 1 are zeros as well.

(b) (3pts each)Determine which matrices are in reduced row echelon form, row echelon form or neither. Justify your answer.

1. $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right].$

   Solution: Row echelon form. The leading 1 are where they are supposed to be but there are entries above the leading once.

2. $\begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$

   Solution: Neither. This matrix is not even in row echelon form. For a matrix to be in row echelon form, any row of all zeros must be on the bottom. The row echelon form of this matrix is $\begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$

5.
(a) (7pts) Find the points of intersection, if any, of the following planes.

$$\begin{aligned} x + 2y - z &= 1 \\ -2x - 3y + 2z &= -1 \\ -5x - 8y + 5z &= 0 \end{aligned}$$

Solution:

$$\textit{augmented} \quad \left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ -2 & -3 & 2 & -1 \\ -5 & -8 & 5 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2' = R_2 + 2R_1 \\ R_3' = R_3 + 5R_1 \end{array} \quad \left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 5 \end{array}\right]$$

$$\begin{array}{l} R_1' = R_1 - 2R_2 \\ R_3' = R_3 - 2R_1' \end{array} \quad \left[\begin{array}{rrr|r} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

Although we haven't moved all the way to reduced row echelon form, we can stop here. Since the row rank of the coefficient matrix does not equal the row rank of the augmented matrix, we can see that this system does not have a solution.

(b) (3pts) Identify the intersection of the planes as the empty set, a point, a line, or a plane. Justify your answer.

Solution: Since there is no solution points, the intersection of the planes is the empty set.

6. Consider the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & 4 \end{array}\right].$$

(a) (4pts) Determine the solution for this system.

Solution: Since the column corresponding to the third variable does not have a leading 1 in it, we set that variable equal to a parameter. Let $z = t$. (Equivalently, $x_3 = t$.) Then the rows of the augmented matrix can be read as

$$\begin{aligned} x + 2t &= 4 \\ y - t &= 4 \end{aligned}$$

Solving these equations for x and y we get the solution set $x = -2t + 4$, $y = t + 4$, and $z = t$ for any t. Equivalently, the solution set is $(-2t + 4, t + 4, t)$ for any t.

(b) (6pts) Assume that this matrix is the end result of a business model you are solving. Let each variable represent a different product your company produces. Using the fact that all of the variables must be non-negative and you need to sell whole items, find all possible solution points.

Solution: $x \geq 0$ implies that $-2t + 4 \geq 0$. Solving this inequality for t, we get $t \leq 2$.

$y \geq 0$ implies that $t + 4 \geq 0$. Solving this inequality for t, we get $t \geq -4$.

Lastly, $z \geq 0$ implies that $t \geq 0$

Combining these constraints on t, we get the t range

$$0 \leq t \leq 2.$$

Since the variables have to take an integer value, t has to take an integer value. This yields only three possible choices for t: $t = 0$, $t = 1$, and $t = 2$. Inserting these values of t into the solution point, we get the following set of solutions:

$$\begin{aligned} t = 0 &\rightarrow (4, 4, 0) \\ t = 1 &\rightarrow (2, 5, 1) \\ t = 2 &\rightarrow (0, 6, 2) \end{aligned}$$

7. Consider the linear program:

$$\begin{array}{ll} \text{maximize} & z = -2x + y \\ \text{subject to} & x + y \le 4 \\ & x - y \le 2 \\ & y \le 2 \\ & x \ge 0, y \ge 0 \end{array}$$

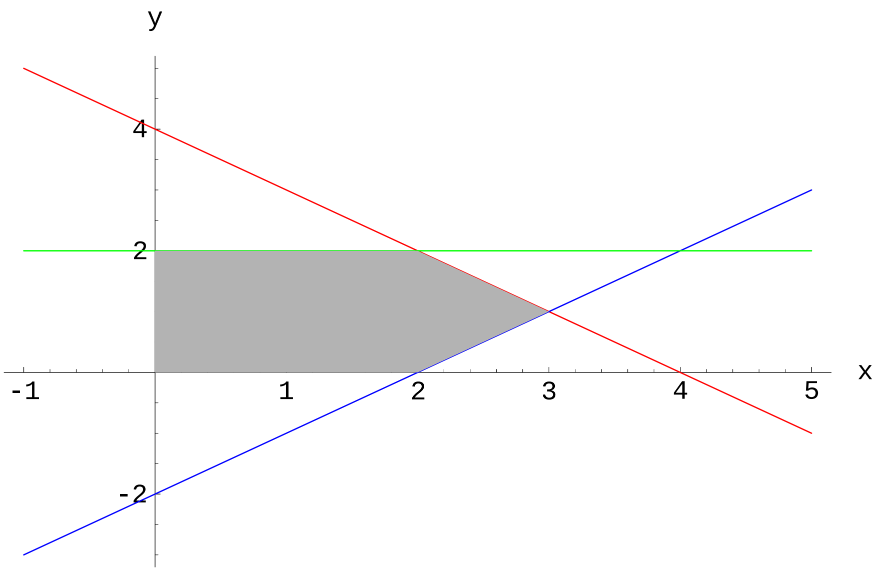
(a) (6pts) Graph the feasibility region and find the corner points.

Solution: Considering the line $x + y = 4$, or $y = -x + 4$ in slope-intercept form, we get the intersection points $(0, 4)$ and $(-4, 0)$. Testing the point $(0, 0)$ in the inequality (i.e. $0 - 0 = 0 \le 4$ is true), we see that the half-plane defined by this inequality is the region below the line $y = -x + 4$.

Similarly, graphing the line $y = x - 2$ using the intercepts $(0, -2)$ and $(2, 0)$ and testing the point $(0, 0)$ in the inequality, we see that the half-plane defined by this inequality is the region above the line $y = x - 2$.

Lastly, we graph the horizontal line $y = 2$. $y \le 2$ defines the half-plane below the line $y = 2$.

Combining these inequalities with the restriction that x and y are non-negative, we get the following polygonal region.



To find the corner points, we recognize three of them instantly: $(0, 0)$, $(0, 2)$ and $(2, 0)$. The intersection of the horizontal line $y = 2$ and the line $y = -x + 4$ happens at $(2, 2)$. The intersection of the line $y = x - 2$ and the line $y = -x + 4$ occurs at $(3, 1)$.

(b) (4pts) Solve the linear program.

Solution: Since we already have the corner points of the polygonal feasibility region, the easiest way to solve the linear is to evaluate z at the corner points.

$$\begin{aligned}
z(0,0) &\rightarrow z = -2(0) + (0) = 0 \\
z(0,2) &\rightarrow z = 2 \\
z(2,2) &\rightarrow z = -2 \\
z(3,1) &\rightarrow z = -5 \\
z(2,0) &\rightarrow z = -4
\end{aligned}$$

The conclusion is that the maximum z value is 2 and occurs at the point $(0,2)$.