Section 1.4: Linear Programming in the Plane

Definition: *a linear function*

Example 1: Consider the feasibility region for the shingle company from Lesson 1.3, Example 3. Now suppose each bundle of shingles sells for \$10. Then the revenue R can be written as a linear function of x and y. Specifically, $R = 10x + 10y$ where x is the number of the first kind of shingles sold and y is the number of the second kind of shingles sold.

Note that different values of R yield different lines with the exact same slope.

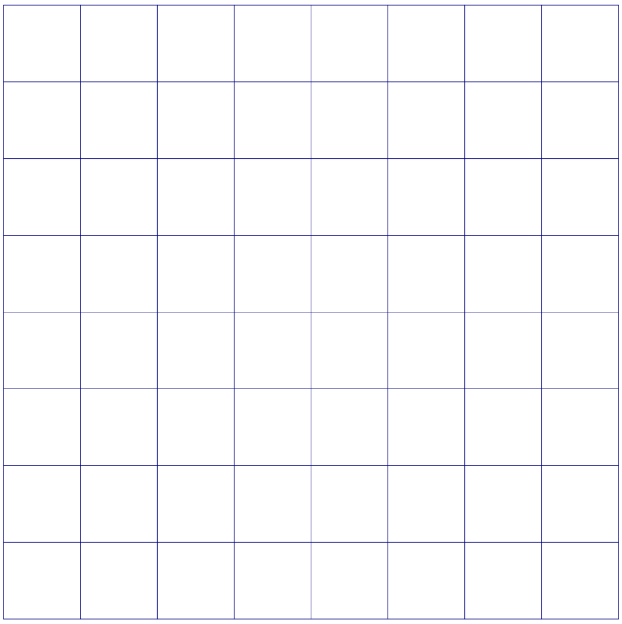
Choosing different values of R, graph some of the lines for $R = 10x + 10y$ over the feasibility region from the shingle company example. Can you draw any conclusions about what type of shingles to produce?

Definition: *lines of constancy*

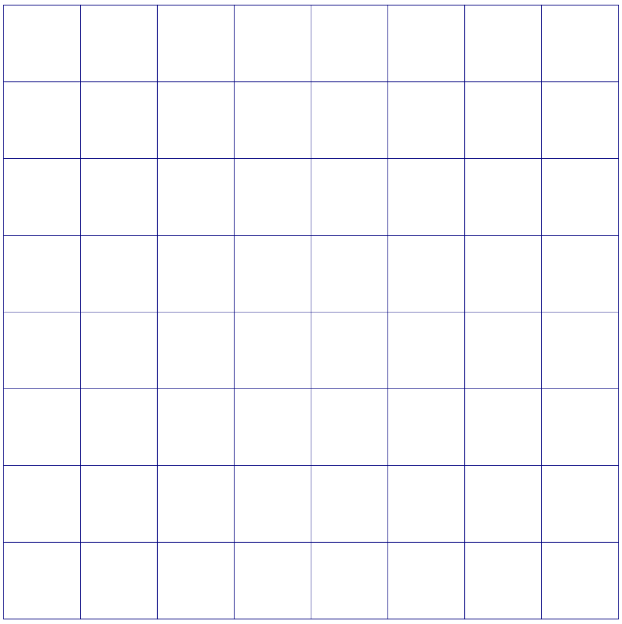
Definition: *a linear program*

Example 2: Solve the linear program using lines of constancy:

$$\begin{aligned} \text{minimize} \quad & z = -2x + y \\ \text{subject to} \quad & x - y \geq -3 \\ & x + y \leq 5 \\ & x \geq 0 \\ & y \geq 0. \end{aligned}$$



Example 3: Find the maximum and minimum values of the linear function $z = -x + 3y$ over the following rectangular region.



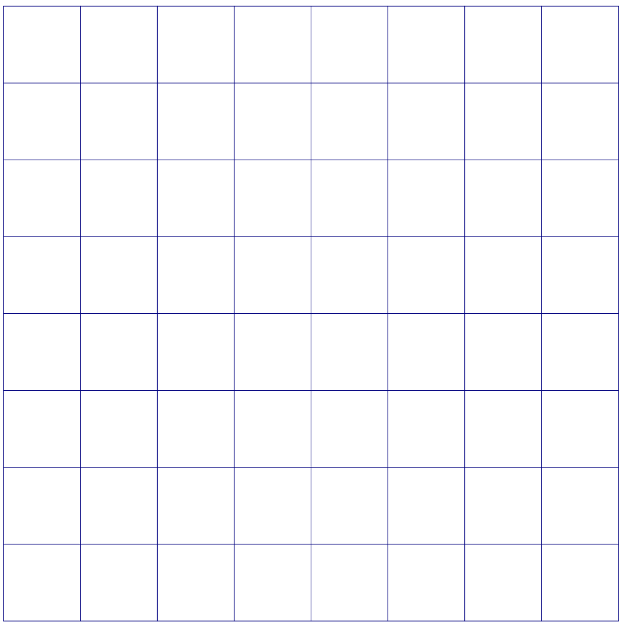
Definition: *a polygon*

Theorem: Let $z = ax + by$ be a linear function, and let P be a polygon in the plane. Then the maximum and minimum values of z are attained at corner points of P.

Procedure for solving a linear program for a polygonal feasibility region:

Example 4: Solve the linear program:

$$\begin{aligned} \text{minimize} \quad & z = -2x + y \\ \text{subject to} \quad & x - y \geq -3 \\ & x + y \leq 5 \\ & x \geq 0 \\ & y \geq 0 \end{aligned}$$



Example 5: Solve the linear program:

$$\begin{aligned} \text{maximize} \quad & z = x - 3y \\ \text{subject to} \quad & 7x + 2y \leq 14 \\ & -3x + y \leq 3 \\ & y \geq 0 \end{aligned}$$