

Math 231 B,C. Fall 2009. Homework from lecture on 8/26.

INSTRUCTIONS. On every homework this semester, you must put a box around each answer. This helps the graders do their job. If you do not put the answer in a box, you will not get credit.

1. Let $f(x) = 2x^2 + x + 1$.

(a) Find the Taylor polynomial of degree 2 about the point $a = 1$.

(b) Multiply out $T_2(x)$. Describe what you find, in one short sentence.

2. (a) Find the linear (first order) approximation to $f(x) = \sin x$ about the point $a = \pi/3$.

(b) Use your answer to approximate the value of $\sin(1)$. How close is your answer to the actual value of $\sin(1)$? [Remember to use radians on your calculator.]

(c) Repeat using the third order approximation to $f(x) = \sin x$ about the point $a = 0$. Is this answer better or worse than part (b)?

3. Approximate $e = e^1$ using the first few terms of the Taylor series about 0.

How many terms do you need, in order to obtain 5 correct decimal places of e ?

4. Find the Taylor series for $f(x) = \sin x$ about $\pi/2$.

5. Use algebraic manipulation and known Taylor series from class to write down Taylor series about 0, for the following functions.

(a) $f(x) = \cosh(x)$ [check: cosh is an even function, so you expect only even powers of x]

(b) $f(x) = \sin(4x)$

(c) $f(x) = e^{-x^2}$

(d) $f(x) = \frac{\sin x}{x}$

6. One engineer says to another:

“whenever I see $\sqrt{1+x}$ in a formula, I replace it with $1 + \frac{1}{2}x$.”

(a) Explain why the engineer does this, in one short sentence.

(b) Using a calculator or a graph, find a number $\delta > 0$ such that if $|x| < \delta$ then the difference between $\sqrt{1+x}$ and $1 + \frac{1}{2}x$ is less than 0.1 in magnitude.