Fall 2008, MWF 9am, Altgeld Hall 243

Math 595: Introduction to Culler-Vogtmann's Outer Space

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Culler-Vogtmann's Outer Space was introduced by Culler and Vogtmann in a seminal paper "Moduli of graphs and automorphisms of free groups". Invent. Math. 84 (1986), no. 1, 91–119. Since then the Outer Space became a central tool in the study of the outer automorphism group of a free group. The Outer Space $CV(F)$ of a finitely generated free group $\mathbf{F}$ is a free group analogue of the Teichmuller space of a Reimann surface. Instead of marked hyperbolic structures, the points of Outer Space are represented by "marked metric graph structures" on a free group $\mathbf{F}$, that can also be thought of as minimal free and discrete isometric actions of $\mathbf{F}$ on R-trees. The Outer Space $CV(F)$ comes equipped with a natural action of $Out(F)$ and the quotient $CV(F)/Out(F)$ is a certain moduli space consisting of finite metric graphs. Unlike the Teichmuller space, the Outer Space of a free group is not a manifold, and its global structure is more difficult to understand. The Outer Space $CV(F)$ is a useful tool in understanding algebraic and dynamical properties of $Out(F)$ and of individual automorphisms of free groups. There is also a natural compactification $\widehat{CV}(F)$ of $CV(F)$ analogous to the Thurston compactification of the Teichmuller space.

We will discuss the basic properties of $CV(F)$ including several equivalent topologies, finite dimensionality, contractibility and implications for computing various homotopy invariants of $Out(F)$. We will also discuss the construction and the properties of $\widehat{CV}(F)$ and its characterization in terms of very small actions of $\mathbf{F}$ on R-trees. Time permitting, we will cover some results related to train-tracks for free group automorphisms and the general structure theory for group actions on R-trees. We will also discuss some other naturally associated structures, such as the space of algebraic laminations on a free group, the space of geodesic currents on a free group, and the notion of a geometric intersection number between points of the Outer Space and geodesic currents.

Prerequisites:

While there are no specific prerequisites for this course, some basic knowledge of algebraic topology and some familiarity with the notion of a free group and the notions of free products and amalgamated free products of groups are expected.

Required Text: There is no required textbook for the course. For the most part we will use several research articles as basic sources.

Recommended Texts:

(1) Ian Chiswell, Introduction to A-trees. World Scientific Publishing Co., Inc., River Edge, NJ, 2001

(2) Allen Hatcher, Algebraic topology. Cambridge University Press, Cambridge, 2002.