Spring 2010

MATH 595 – DYNAMICS ON NETWORKS

12-12:50 MW, 447 Altgeld Hall

Instructor: Eugene Lerman

CRN 53838

The subject of the course is dynamics on networks as developed by M. Golubitsky, I. Stewart and their collaborators and summarized in a highly influential paper "Nonlinear dynamics of networks: the groupoid formalism," Bull. Amer. Math. Soc. **43** (2006), 305-364.

The course will mix ideas from dynamical systems and category theory. No background in category theory is assumed. It would be useful to have a nodding acquaintance with (geometric) ODEs and some basic group theory, but any gaps in the background can be overcome with sufficient work.

The course grade will be based on in-class reports. Students interested in pursuing the subject further may undertake a research project over the summer (REGS).

Networks are ubiquitous in science and engineering. They model complex systems with many weakly interacting subsystems. To first approximation one can model a network by a directed graph with the nodes representing the subsystems and directed edges standing for the interactions. A more detailed model assigns to each node a phase space. The total phase space of the complex system is the product of the phase spaces assigned to the nodes.
The admissible dynamics on the total phase space is governed by an ODE, and the directed graph governs the dependence of various components of this ODE on the constituent subsystems. The admissible ODEs posses a subtle symmetry discovered by Golubitsky et al. called groupoid invariance. This symmetry generically gives rise to synchrony.