

Math 595 LP (Spring 2010)

Stable processes and its Potential theory

- **Instructor:** Panki Kim (331 Illini Hall)
- **Text:** No text, lecture note will be distributed in class.

General Description:

There are important and beautiful connections between Probability and classical potential theory. Such connection can be extended to a large class of Markov processes. This course will treat the fundamentals of symmetric stable process and its potential theory. Our principal aim is to cover some of recent developments of this topic.

Course topics:

1. Elements of Probability Theory.
2. Review on Levy process
3. Symmetric Stable process
4. Symmetric Stable process in open subsets
5. Fractional Laplacian and its harmonic function
6. Properties of harmonic function with respect to Fractional Laplacian
7. Intrinsic Ultracontractivity
8. Dirichlet Heat Kernel Estimates

Prerequisite: Graduate probability and measure theory (Math540 and Math561)

References;

1. K. Bogdan, T. Byczkowski, T. Kulczycki, M. Ryznar, R. Song and Z. Vondracek: Potential analysis of stable processes and its extensions.
2. Z.-Q. Chen, Renming Song, Intrinsic ultracontractivity and conditional gauge for symmetric stable processes. *J. Funct. Anal.*, Vol. 150 (1997), 204-239.
3. Z.-Q. Chen, Renming Song, Estimates on the Green functions and Poisson kernels of symmetric stable processes. *Math. Ann.*, Vol. 312 (1998), 465-501.
4. Z.-Q. Chen and R. Song: Martin boundary and integral representation for harmonic functions of symmetric stable processes. *J. of Funct. Anal.*, Vol. 159 (1998), 267-294.
5. Z.-Q. Chen, P. Kim and T. Kumagai. Notes on heat kernel estimates and parabolic Harnack inequality for jump processes.
6. Z.-Q. Chen, P. Kim and R. Song: *Heat kernel estimates for Dirichlet fractional Laplacian (to appear in J. European Math. Soc.)*
7. R. K. Gettoor, First passage times for symmetric stable processes in space, *Trans. Amer. math. Soc.* 101 (1961), 75-90.
8. T. Kulczycki, Intrinsic ultracontractivity for symmetric stable processes, *Bul l. Polish Acad. Sci. Math.*, 46 (1998), 325–334.