

MATH 595 — Methods of Applied Mathematics

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Who should take this course. This course will be a thorough introduction to the modern methods of applied mathematics. It is intended to appeal to two groups:

- Graduate students in **Mathematics** who are interested in working on applied problems and connecting their mathematical knowledge to real scientific and engineering applications, and
 - Graduate students in **Science** and **Engineering** who use (or will use) mathematical tools in their work who are interested in learning the general theory of these tools and the mathematical context which gives these tools deeper meaning.
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Course overview. “Applied mathematics” is a rich, broad, and deep field which uses results from many branches of pure mathematics and connects them to problems in science and engineering. The field could not be comprehensively covered in one semester (or perhaps even in one lifetime). What we will do instead is introduce several of the powerful perspectives which make up modern applied mathematics and, for each, present one or more important applications. We will cover both exact mathematical analysis and computational techniques.

In particular, this course will be very useful for graduate students who are interested in eventually doing research in applied mathematics. Each topic will include an overview of the open questions and “Big Problems” in the field, and a list of resources of where to look next for students interested in the topic.

The topics will include:

1. Nondimensionalization and scaling analysis
 - **Applications:** Critical phenomena in sandpiles and in earthquakes; Kolomogorov’s 5/3 scaling in turbulence
 2. Regular asymptotics — mean-field laws, mass-action laws, Large Number limits for large systems, van Kampen expansions, Kramers-Moyal expansions, Gronwall’s estimates
 - **Applications:** epidemic sizes in large populations; short-time orbital mechanics; homogeneous steady-states in networks
 3. Singular asymptotics — multiscale systems, homogenization, WKB expansions, boundary layers, traveling waves in reaction–diffusion systems, large deviations
 - **Applications:** neural models (Hodgkin–Huxley, Fitzhugh–Nagumo) and their dynamics; switching times between equilibria for biochemical and biological systems; epidemics in small populations; semiclassical approximations in quantum mechanics; dynamics of excitable media
 4. Complex systems — spectral theory, stability analysis
 - **Applications:** scale-free networks (in social networks, epidemiology, computer networks); synchronization and critical phenomena for biophysical systems
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Prerequisites. This course will require undergraduate background in ODEs, PDEs, probability theory (MATH 441, 442, 461 or equivalents). However, the more a student brings to the course, the more the student will get out of it, so graduate courses in these areas can only help. Interested students who are not sure if they have sufficient background are encouraged to email me at rdeville@illinois.edu and discuss their readiness.