

**Math 130 E. Notes on Series and Information on Exam 2. March 30, 2004**  
(These notes are meant to be helpful but not comprehensive)

Start with a series  $\sum a_n$ . The basic question is: Does it converge?

- Does  $\lim a_n = 0$ ? If the answer is NO, then the series diverges by the Test for Divergence. If the answer is YES, then the series may converge or diverge.

- Is the series alternating? If the series has the form  $\sum(-1)^n a_n$  or  $\sum(-1)^{n+1} a_n$  with positive  $a_n$ , then try the Alternating Series Test: Check that

- 1)  $\lim a_n = 0$  and that

- 2)  $a_n$  is decreasing (i.e.  $a_{n+1} \leq a_n$  from some point on).

If (1) and (2) hold then the AST tells us that the series converges.

- Does the series involve factorials and terms of the form  $r^n$  with constant  $r$ ? Then the Ratio Test may be a good bet. If possible, evaluate the number

$$\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

If  $\rho < 1$  then the series converges absolutely. If  $\rho > 1$  then the series diverges. If  $\rho = 1$  then the test is inconclusive.

Note: This only tests for absolute convergence.

- Does the series have terms of the form  $f(n)^n$  with functions  $f(n)$ ? If so then the Root Test may work.

If possible, evaluate the number

$$\rho = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}.$$

If  $\rho < 1$  then the series converges absolutely. If  $\rho > 1$  then the series diverges. If  $\rho = 1$  then the test is inconclusive.

Note: This only tests for absolute convergence.

- Important series are: the  $p$ -series  $\sum \frac{1}{n^p}$ , which converges if  $p > 1$  and diverges if  $p \leq 1$ . Also the geometric series  $\sum r^n$ , which converges if  $|r| < 1$  and diverges if  $|r| \geq 1$ .

- The Comparison Test and Limit Comparison Test can be used to test convergence of series of positive terms which are “similar” to known series (e.g. a  $p$ -series or geometric series).

The Limit Comparison test is useful when the terms are rational functions; i.e. the series has the form

$$\sum \frac{an^k + bn^{k-1} + \text{lower terms}}{cn^j + dn^{j-1} + \text{lower terms}}.$$

The terms “look like” they have size  $\frac{n^k}{n^j} = \frac{1}{n^{j-k}}$ . So we use the Limit Comparison Test with the  $p$ -series  $\sum \frac{1}{n^{j-k}}$ . This also works when the terms involve roots and powers. In each case, you must identify the “main term” in order to choose the  $p$ -series which you should use for the comparison.

Note: The ratio test will always be inconclusive if the terms are rational functions!

The Comparison Test is useful when

- 1) The terms  $a_n$  are smaller than terms  $b_n$  in a series which is known to converge, or

- 2) The terms  $a_n$  are bigger than terms  $b_n$  in a series which is known to diverge.

- The Integral Test is useful when the terms  $a_n$  are positive and have the form  $a_n = f(n)$ ,

where the improper integral  $\int_1^{\infty} f(x) dx$  can be computed.

## Things to know for the exam

- (1) Exam covers Section 8.1 and Chapter 10 (through 10.9).
- (2) All basic definitions and concepts. **There will be short-answer questions on these topics which test your basic knowledge.** See the Chapter Review: Definitions, Concepts, Results (page 769) for a good list of topics to consider. A sample question would be “Define what it means for the infinite series  $\sum_{n=1}^{\infty} a_n$  to converge to  $S$ .”
- (3) Sequences.
- (4) Taylor and Maclaurin Series and polynomials.
- (5) Statements and use of all tests for convergence. Absolute and conditional convergence.
- (6) Power Series (Section 10.8 and 10.9). The amount of Section 10.9 to be covered on the exam will be announced next week.
- (7) More practice problems for the exam. Chapter review problems, old homework, and quiz problems are also recommended. Links to solutions will be available from the course homepage.

<http://www.math.uiuc.edu/~ahlgren/math130/math130.html>

10.2: 3, 4, 6, 8, 15, 16, 21, 33, 35, 39

10.3: 1, 9, 15, 22, 27, 35, 46, 52, 54

10.4: 3, 5, 13, 15, 23, 26, 27, 31, 33, 38, 52, 54

10.5: 5, 10, 15, 22, 29, 30, 31, 32, 35, 37

10.6: 5, 10, 15, 20, 25, 30, 35, 40, 42

10.7: 5, 8, 17, 21, 28, 34, 39, 40, 43, 44

10.8: 5, 8, 11, 17, 20, 29, 30, 31, 33, 39, 43, 46, 49, 50, 51, 52, 57, 58, 59.

- The exam is on Thursday, April 8 from 7-8:15 pm. You may stay until 8:30 if you wish.

The location of your exam is determined as follows:

8 am, 9 am, 1 pm sections: 151 Loomis

11 am, 12 pm, 3 pm sections: 314 Altgeld Hall

- **Conflict Exams.** If you need a conflict exam, you must do the following:

1) Email me **by Friday, April 2 at 5 pm**. Use the single word **Conflict** as the subject of your email.

2) Indicate the nature of your conflict.

3) Your preference for conflict time. Possibilities are 1) Before the regular exam (Thursday at 5 pm). 2) Wednesday evening ( 5 pm or 7 pm).

Please indicate which of these times you cannot make.

4) Location and time of conflict exam to be announced.