Math 285 — Midterm 3 practice solutions

Problem 1: This is a typical Sine Fourier series calculation (with $L = 2$). The result is

$$\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{2}\right)$$

where

$$b_n = \begin{cases} -\frac{8}{n\pi} \text{ for n even} \\ \frac{8}{n\pi} - \frac{32}{n^3\pi^3} \text{ for n odd} \end{cases}$$

Problem 2: Given the forcing to this oscillator, we look for a solution of the form:

$$y(x) = \sum_{n=1}^{\infty} B_n \sin(nx)$$

which already satisfies the boundary conditions. Using the given equation we get:

$$B_n = \frac{1}{n(\frac{1}{25} - n^2)}$$

Notice that $y_c = 0$ because of the boundary conditions.

Problem 3: This is a typical diffusion problem. We solve it using separation of variables to obtain the solution:

$$y(x,t) = 4 + 3\,\mathrm{e}^{-3\pi^2 t} \cos(\pi x)$$

The coefficients 4 and 3 can be obtained by inspection after imposing the initial condition.

Problem 4: This is a straightforward wave equation problem, solved with separation of variables to get:

$$y(x,t) = \frac{1}{2\pi}\sin(2\pi t)\,\sin(\pi x) - \frac{1}{6\pi}\sin(6\pi t)\,\sin(3\pi x)$$

Again, the coeffcients can be obtained by inspection after imposing the two initial conditions.