Math 285 — Midterm 1 practice solutions

Problem 1: This is a linear homogeneous constant coefficient ODE. To solve it we try a solution of the form $y = e^{rx}$ and get the characteristic equation for r:

$$r^2 + 2r + 6 = 0$$

which has solutions $-1 \pm \sqrt{5}\,i$. So the general solution is of the form:

$$y = e^{-x}\left[A\cos(\sqrt{5}x) + B\sin(\sqrt{5}x)\right]$$

Using the initial conditions we get the solution

$$y = \frac{2}{\sqrt{5}} e^{-x}\sin(\sqrt{5}x)$$

Problem 2: This is a simple integrable ODE. After integrating three times and applying the initial conditions we get the solution:

$$u(t) = -e^t + \frac{3}{2}t^2 + t + 1$$

Problem 3: This population equation has two equilibrium solutions: a stable one at $P = 2$ and an unstable one at $P = 3$. The slope field would show solutions above $P = 3$ increasing exponentially; solutions between $P = 3$ and $P = 2$ moving away from $P = 3$ and approaching $P = 2$ asymptotically, and solutions below $P = 2$ increasing and approaching $P = 2$ asymptotically. The population with $P(0) = 4$ grows with time and explodes at finite time. The analytic solution is given by:

$$P(t) = \frac{6 - 2e^t}{2 - e^t}$$

Problem 4: This is a first order linear equation, so we can use an integrating factor. The integrating factor would be $e^{t^2/2}$. We can then solve the equation and after using the initial condition we get:

$$x(t) = \left(\frac{t^2}{2} - 1\right) \mathrm{e}^{-t^2/2}$$

Problem 5: One way to solve this equation is ti use the substitution $v(x) = 2x + y(x)$. The resulting equation for $v(x)$ is

$$\frac{\mathrm{d}v}{\mathrm{d}x} = \sqrt{v}$$

which is separable. The final solution for $y(x)$ is

$$y = \frac{(x+c)^2}{4} - 2x$$