

385 Differential Equations and Orthogonal Functions

Assignment 1-solutions

1.2.44

Let $a > 0$ denote the constant deceleration of the car when braking, and take $x_0 = 0$ for the cars position at time $t = 0$ when the brakes are applied. In the police experiment with $v_0 = 25$ ft/sec, the distance the car travels in t seconds is given by

$$x(t) = -\frac{1}{2}at^2 + \frac{88}{60}25t$$

(with the factor $\frac{88}{60}$ used to convert the velocity units from mi/hr to ft/s). When we solve simultaneously the equations $x(t) = 45$ and $x'(t) = 0$ we find that $a = \frac{1210}{81} \approx 14.94$ (ft/s)/s. With this value of the deceleration and the (as yet) unknown velocity v_0 of the car involved in the accident, its position function is

$$x(t) = -\frac{1}{2} \frac{1210}{81} t^2 + v_0 t.$$

The simultaneous equations $x(t) = 210$ and $x'(t) = 0$ finally yield $v_0 = \frac{110}{9} \sqrt{42} \approx 79.21$ ft/sec, almost exactly 54 mph.