385 Differential Equations and Orthogonal Functions
Exam 1

Total points: 35 (=7 $\times$5 points). Do all questions. Explain all answers. No notes, books, calculators or computers are allowed.

To answer the questions you may use the back of the paper.

1. Find all solutions of the differential equation
$\cos x + \ln y + (\frac{x}{y} + e^y)\frac{dy}{dx} = 0.$

2. Find all solutions of the differential equation
$x^2y' = xy + x^2e^{y/x}$.

3. Find the solution of the initial value problem
$y' + y = e^x, y(0) = 1.$

4. Determine whether there exists a unique solution of the given initial value problem.

a) $\frac{dy}{dx} = x \ln y, y(2) = 2$
b) $\frac{dy}{dx} = \sqrt{x - y}, y(2) = 1$
c) $y\frac{dy}{dx} = x - 1, y(2) = 0.$

5. Write $x(t)$ for the height at time t of an object that is falling downward under the influence of gravity. Assume the object encounters air resistance proportional to its velocity $v(t) = \frac{dv}{dt}$. Then by Newton's Law we get

$$\frac{dv}{dt} = -kv - g$$

(for some positive constants k and g).
a) Solve for $v(t)$.
b) Does the velocity of the object increase indefinitely or does it approach a finite limiting speed? Explain briefly.

6. Suppose that when a certain lake is stocked with fish, birth rate $\beta(t)$ and death rate $\delta(t)$ are both proportional to $\frac{1}{\sqrt{P}}$. Use the general population equation

$$\frac{dP}{dt} = (\beta - \delta)P$$

to answer the following questions:
a) Show that

$$P(t) = (\frac{1}{2}kt + \sqrt{P_0})^2,$$

where k is a constant.
b) If $P_0 = 100$ and after 6 months there are 169 fish in the lake, how many will there be after 1 year?

7. Consider the differential equation

$$xy + y^2 - x^2 \frac{dy}{dx} = 0.$$

a) Transform this equation in a form covered in the lecture. Which type if differential equation is it?

b) Solve this differential equation.