

385 Differential Equations and Orthogonal Functions

Exam 2

Total points: 32. Do all questions. Explain all answers. No notes, books, calculators or computers are allowed.

To answer questions, you may use the back of the paper.

1. We consider the nonhomogeneous differential equation

$$y^{(5)} - 5y^{(4)} + 9y^{(3)} - 5y^{(2)} = x + e^x.$$

a) (3 points)

As a first step, write down the associated homogeneous differential equation and its general solution.

Hint: You may use (**without calculation**) that the roots of the polynomial equation

$$r^5 - 5r^4 + 9r^3 - 5r^2 = 0$$

are at $0, 0, 1, (2 \pm i)$.

b) (3 points)

Write down the trial solution you would substitute in the differential to solve the nonhomogeneous differential equation (**you do not have to solve the nonhomogenous differential equation**).

2. (4 points) The differential equation

$$y^{(3)} + 4y' = 0$$

has the solutions $y_1(x) = 1$, $y_2 = (\sin x)^2$ and $y_3 = (\cos x)^2 + 2$ on the open interval $(1, 2)$. Is this information sufficient to give the general solution of this homogeneous differential equation? Explain your answer.

3. (4 points) Is the value $\lambda = -1$ an eigenvalue of the endpoint value problem

$$y'' + \lambda y = 0, y(0) = 0, y(1) = 0$$

on the real axis? Explain your answer.

4. (4 points) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' - 4y' + 4y = e^{2x}.$$

5.

A body with mass $m = 0.5$ kg is attached to a spring (spring constant $k = 50$ N/m) and a dashpot (damping constant $c = 1$ Ns/m). The position function of the body is described by the differential equation

$$mx'' + cx' + kx = 0.$$

a) (4 points)

Calculate the position function $x(t)$.

b) (2 points)

Decide whether the body approaches its equilibrium position $x = 0$. Explain your answer.

6. We consider an undamped forced motion of a body with mass $m = 1$ kg attached to a spring ($k = 100$ N/m) and an external force $F(t) = \cos 10t$. The position function of the body is described by the differential equation

$$mx'' + kx = F(t).$$

a) (4 points)

Solve the resulting differential equation for the position function $x(t)$.

b) (2 points)

Is this an example of resonance? Explain your answer.

c) (2 points)

Briefly explain the phenomenon of resonance.