446 Complex Analysis and its Applications
Exam 2

Total points: 36 (=6 ×6). Do all questions. Explain all answers. No notes, books, calculators or computers are allowed. To answer questions, you may use the back of the paper.

1. Calculate the value of the integral

$$\int_C \frac{e^z}{z^2(z+1)^3} dz,$$

where

a)

the contour C is given by $z(t) = \frac{1}{2}e^{it}, 0 \leq t \leq 2\pi$

b)

the contour C is given by $z(t) = 2e^{it}, 0 \leq t \leq 2\pi$.

2. Consider

$$f(z) = \frac{-2}{(z-2)(z-4)} = \frac{1}{z-2} - \frac{1}{z-4}.$$

a)

What are the maximal disks or annuli centered at 0 where a Taylor series expansion or Laurent series expansion of f about 0 exists?

b)

Calculate the expansions in all domains you listed in a).

3.
Find the value of the integral of $f(z)$ around the circle $|z-i| = 3$ taken in the positive sense when

$$f(z) = \frac{1}{(z+3i)(z-3i)}.$$

4. The following functions have isolated singular points at 0. Are these isolated singular points removable singularities, poles or essential singularities? If there are poles, determine the order.

a)

$$e^{1/z^2}$$

b)

$$\frac{\sin z}{z}$$

c)

$$\frac{e^z - 1}{z^2}.$$

5.

a)

Find a parametric representation $z(t)$ of a square with vertices $1-i, 1+i, -1+i, -1-i$. The square should be positively oriented.

b)

Let $z(t)$ be the parametric representation of the square C in a), and let f be a function that is analytic on and in the exterior of C.

Let C_1 be given by $z_1(t) = \sqrt{2}e^{-it}, 0 \leq t \leq 2\pi$.

If

$$\int_C f(z)dz = \omega,$$

what is

$$\int_{C_1} f(z)dz?$$

6. Find the Taylor Series about 0 of

$$f_1(z) = \sin z,$$

$$f_2(z) = \frac{1}{(2-z)^2}$$

and

$$f_3(z) = \frac{\sin z}{(2-z)^2}.$$