484 Nonlinear Programming
Exam 1

Total points: 60(=6 $\times$ 10) points. Do all questions. Explain all answers. No notes, books, calculators or computers are allowed.

1.

a)

Identify the coercive functions in the following list. Explain your answers.

$f(x, y) = x^2 - 3xy + x^2$

$g(x, y, z) = x^2 + y^2$

$h(x, y) = e^{(x-1)^2+y^2}$.

b)

For each coercive function in the list above, determine the global minimizer.

2.
Classify the following matrices according to whether they are positive or negative definite or semidefinite or indefinite.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 5 \end{pmatrix},$$

$$\begin{pmatrix} -1 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 2 & -1 \end{pmatrix}.$$

3.
Show that although (0,0) is a critical point of $f(x_1, x_2) = x_1^5 - x_1 x_2^6$, it is neither a local maximizer nor a local minimizer.

4.
Use an appropriate convex function to find all global minimizers of

$$f(x,y,z) = \left(\frac{x^4+y^4+z^4}{3}\right)^{1/4} - \frac{x+y+z}{3}.$$

5.
Let (GP) be the following primal geometric program:
Minimize

$$g(t) = t_1 + \frac{1}{t_2} + t_2 + \frac{1}{t_2^2}, t_1 > 0, t_2 > 0.$$

a) Write down the dual geometric program.
b) Compute the set F of feasible vectors.
c) Write down the function in one variable that has to be maximized in order to solve the dual program (do not forget to give the interval on which the function has to be maximized).

6.

Use the principal minor criteria to determine the nature of the critical point of the following function:

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1x_2.$$