

## 484 Nonlinear Programming

### Exam 2

1.

Let  $f(x) = x^4 - 8x^2$ .

a) Calculate  $x^{(1)}$  and  $x^{(2)}$  of Newton's method for minimizing functions with starting value  $x^{(0)} = 1$  and  $0.1$ .

b) Give three possible limits of the Newton's method for  $f$  and the nature of these points (maximizer, minimizer, saddle point).

2. Let  $f(x) = x^4 - 8x^2$ .

The modified Newton's method is defined by

$$x^{(k+1)} = x^{(k)} - t_k \frac{f'(x^{(k)})}{f''(x^{(k)})},$$

where  $t_k$  is the minimizer of

$$f(x^{(k)} - t \frac{f'(x^{(k)})}{f''(x^{(k)})}).$$

a) Is this method a descent method? Explain your answer.

b) Can this method converge to points that are not minimizers of  $f$ ? Explain.

3.

Let  $f(x, y) = x^2 + y^2$ .

a) For  $(x^{(0)}, y^{(0)}) = (1, 1)$ , find all descent directions.

b) Calculate  $(x^{(1)}, y^{(1)})$  for the method of steepest descent.

c) For any function  $g$  with continuous partial derivatives, suppose that  $(x^{(0)}, y^{(0)}) = (1, 0)$  and  $(x^{(1)}, y^{(1)}) = (0, 0)$ . What are possible options for the next search direction of the method of steepest descent?

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Compute the first two iterates using Broyden's Method for  $x^{(0)} = (1, 1)$  and

$$f(x_1, x_2) = 2x_1^2 + 2x_2^2 - x_1x_2$$

with  $D_0 = I$ .

5. Let  $f(x, y) = x^4 - 4xy + y^4$  and  $x^{(0)} = (1/2, 1/2)$ .

a) Find a value  $\mu_0$  such that  $Hf(x^{(0)} + \mu_0)$  is positive definite.

b) Set  $p^0 = -Hf(x^{(0)} + \mu_0 I)^{-1} \nabla f(x^{(0)})$ .

c) Find a  $t_0$  such that the conditions (3) and (4) hold for  $x^{(1)} = x^{(0)} + t_0 p^{(0)}$ .