484 Nonlinear Programming- Solutions of Test 2

1.
Using the formula

$$x^{(k+1)} = x^{(k)} - \frac{f'(x^{(k)})}{f"(x^{(k)})},$$

we obtain for $x^{(0)} = 1$: $x^{(1)} = 1, x^{(2)} = -2$,
and for $x^{(0)} = 0.1$: $x^{(1)} = \frac{-2}{3970}, x^{(2)} = \frac{2(\frac{-2}{3970})^3}{3(\frac{-2}{3970})^2-4}$.
b) Possible limits are the critical points, these are $-2, -$ and 2. -2 and 2 are global minimizers, 0 is a saddle point.

2.
a)

Yes, it is. If $x^{(0)} > 2$, then $\frac{f'(x^{(k)})}{f"(x^{(k)})} > 0$, so the search direction is "to the left" and $x^{(0)}$ is a global minimizer. If $x^{(0)} < 2$, then $\frac{f'(x^{(k)})}{f"(x^{(k)})} < 0$, so the search direction is "to the right" and $x^{(0)}$ is a global minimizer as well. For any other starting value that is not a critical point, the next iterate is a global minimizer no matter what the search direction is, so in any case the method is descent.

b)

If $x^{(0)} = 0$, then $x^{(k)} = 0$ for all k, sothe method does not converge to a minimizer in this case. For any other starting value the method does converge to a global minimizer.

3.
The descent directions are all directions p with $p\nabla f((1,1)) < 0$.
$\nabla f((1,1)) = (2,2)$, so this condition means

$$2p_1 + 2p_2 < 0$$

or

$$p_1 + p_2 < 0.$$

b)

$$x^{(1)} = (1,1).$$

c)

The next search direction has to be orthogonal to the previous one. Therefore, it should be $(0, a)$ for some $a \in \mathbb{R} \setminus \{0\}$.

4.

$$x^{(1)} = (-2, -2), x^{(1)} = (0, 0).$$

5.

a)

Any $\mu > 1$ works.

b)

$$p^{(0)} = (-3/2, -3/2).$$

c)

$t = 1/2$ works.