

Practice Exam 1

The date for Exam 1 is Friday, February 20, 9-10 am, in class.

1 Compute

(a) (by substitution) [5 pts]
$$\int_0^{\frac{\pi}{2}} \sin(x)\cos(x)dx =$$

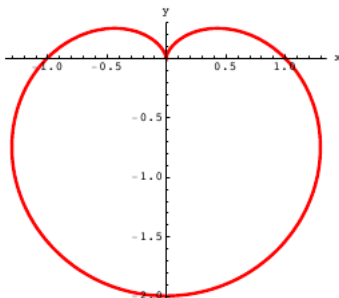
(b) (by parts) [5 pts]
$$\int_1^2 \text{Log}(x)dx =$$

(c) (by using partial fractions) [5 pts]
$$\int_2^3 \frac{1}{x^2-5x+4}dx =$$

(d) (by substitution) [5 pts]
$$\int_1^8 \frac{1}{1+\sqrt[3]{x}}dx =$$

(e) (by using a trigonometric identity) [5 pts]
$$\int_0^{\frac{\pi}{2}} \sin^2(x)dx =$$

2 Consider the curve in the image given by the following parametric equations:



$$\begin{aligned} r(t) &= 1 - \sin(t) \\ x(t) &= r(t)\cos(t) \\ y(t) &= r(t)\sin(t) \\ t_{low} &= 0 \\ t_{high} &= 2\pi \end{aligned}$$

(a) What is the point on the curve corresponding to

- $t = 0$ [1 pt]
- $t = \frac{\pi}{2}$ [1 pt]
- $t = \pi$ [1 pt]

(b) Is the curve traced out clockwise or counterclockwise? [2 pts]

(c) Knowing the area inside the curve is $\frac{3\pi}{2}$, what is the value of [5 pts]

$$\int_0^{2\pi} x'(t)y(t)dt =$$

Why?

(d) Knowing the area inside the curve is $\frac{3\pi}{2}$, what is the value of [5 pts]

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x'(t)y(t)dt =$$

Why?

3 Solve the following differential equation: [15 pts]
 $y'[x] = y[x]$ with $y[0] = 5$.

4 Look at the integral $\int_{-2}^2 \sqrt{4 - x^2} dx$.

(a) Transform the integral by making the substitution $x = 2\sin(u)$. [5 pts]

(b) Simplify it further by using the trigonometric identity $\cos^2(u) = \frac{1 + \cos(2u)}{2}$ [5 pts]

(c) Now compute the integral. [5 pts]

5 Set $Int(n) = \int_0^1 x^n e^{-x} dx$. [10 pts]
Find a formula that expresses $Int(n)$ in terms on $Int(n - 1)$.