Practice Exam 2

The date for Exam 2 is Monday, April 5, 9-10 am, in 241 Altgeld.

1 Determine the first terms (up to x^4) of the power series expansion of the following functions.

(a) $f(x) = sin(3x) * cos(x)$ [5 pts]

(b) $f(x) = \frac{1}{(1-x)^2}$ [5 pts]

2 Give the number that is equal to each of the following infinite sums. Explain what power series you used to get the answer.

(a)

$$1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \ldots + (-1)^n \frac{\pi^{2n}}{(2n)!} + \ldots$$

[5 pts]

(b)

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots + \frac{1}{n!} + \ldots$$

[5 pts]

(c)

$$1 + \frac{1}{2 \cdot 1!} + \frac{1}{2^2 \cdot 2!} + \frac{1}{2^3 \cdot 3!} + \frac{1}{2^4 \cdot 4!} + \ldots + \frac{1}{2^n \cdot n!} + \ldots$$

[5 pts]

3 You are given the following information about two functions $f(x)$ and $g(x)$: The expansions of $f(x)$ and $g(x)$ in powers of $(x-2)$ start like this:

$$f(x) = (x-2)^2 + 5(x-2)^3 + 10(x-2)^4 + \dots$$

$$g(x) = \frac{3}{2}(x-2)^2 + \frac{2}{3}(x-2)^3 + \dots$$

(a) Compute

$$\lim_{x \to 2} \frac{f(x)}{g(x)}.$$

[5 pts]

(b) Find the order of contact of $f(x)$ and $g(x)$ at $x = 2$. Explain your answer. [5 pts]

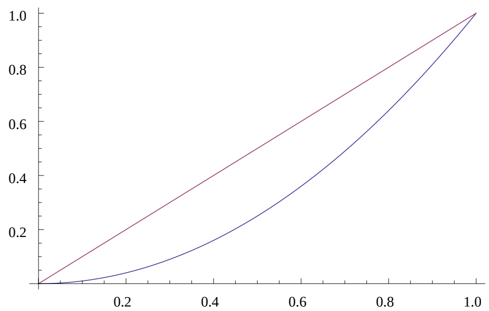
(c) Describe the transition between the graphs of the two functions at $x = 2$. [5 pts]

(d) What will be the order of contact of $f'(x)$ and $g'(x)$ at $x = 2$? How about the order of contact of the antiderivatives? [5 pts]

4 Compute

$$\iint_R e^{3x+4y}\,dx\,dy$$

over the region R that lies between the graphs of the two functions $y = x$ and $y = x^2$. [10 pts]



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5 Let the region R be the circle $(\frac{x}{2})^2 + (\frac{y}{2})^2 = 1$. In the following we shall use Gauss Green to compute the area of this circle.

(a) What should we choose for $f(x, y)$ such that [5 pts]

$$\iint_R f(x, y) dx dy = Area\ of\ R\ ?$$

(b) Using the $f(x, y)$ found before and the formula [5 pts]

$$n(x, y) = \int_0^x f(s, y) ds,$$

find $n(x, y) =$

(c) Now, using the parametrization [5 pts]

$$x(t) = 2Cos(t), y(t) = 2Sin(t), 0 \leq t \leq 2\pi$$

set up (**but do not solve yet**)the Gauss Green simple integration

$$\int_0^{2\pi} n(x(t), y(t))y'(t)dt =$$

(d) Solve the simple integral in part (c). Use the trigonometric formula $Cos(x) = \frac{1}{2}(Cos(2x) + 1)$. [5 pts]

(e) Why does this give you the area of the circle (the region R)? [5 pts]