Practice Final Exam

The Final Exam is Thursday, May 14, 9-11 am, in 241 Altgeld.

Basic Problems

1 (a) Find the first three terms in the expansion of $g(x) = sin(2x)$ in powers of x using basic expansions. [5 pts]

(b) Find the first three terms in the Taylor series expansion of $f(x) = ln(1+x)$ in powers of x. [10 pts]

(c) Find the limit [5 pts]

$$\lim_{x \to 0} \frac{ln(1+x)}{sin(2x)}$$

(d) Give an example of a polynomial that has order of contact 0 with $sin(2x)$ at $x = 0$. [2 pts]

(e) Give an example of a polynomial that has order of contact 2 with $sin(2x)$ at $x = 0$. [1 pts]

(f) Give an example of a polynomial $P(x)$ such that [2 pts]

$$\lim_{x \to 0} \frac{P(x)}{sin(2x)} = 20.$$

2 Use a basic expansion to give the number that is equal to each of the following infinite sums. **Write down the power series you used to get the answer and its convergence interval.**

(a) $1 + \frac{1}{3} + \frac{1}{2!3^2} + \frac{1}{3!3^3} + \ldots + \frac{1}{n!3^n} + \ldots =$ [5 pts]

(b) $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n} + \ldots =$ [5 pts]

(c) $1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \ldots + (-1)^n \frac{\pi^{2n}}{(2n)!} + \ldots =$ [5 pts]

3 Compute the following antiderivatives:

(a)

$$\int_1^e \frac{1}{x}(1+ln(x))^5 dx =$$

[10 pts]

(b)

$$\int_0^1 ln(x)dx =$$

[10 pts]

(c)

$$\int_{-1}^1 \sqrt{1-x^2}dx =$$

[10 pts]

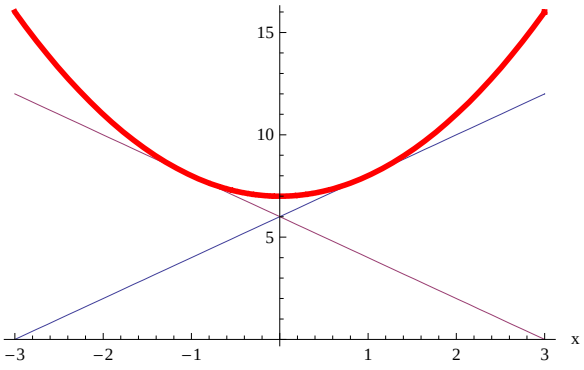
Hint: use a trigonometric substitution.

Advanced Problems

4 Set up and solve a system of equation which insures that a spline

$$f(x) = a + bx + cx^2 + dx^3$$

has order of contact **one** with the line $y = 2x + 6$ at $x = 1$ and order of contact **one** with the line $y = 6 - 2x$ at $x = -1$. [10 pts]



5 Find the **convergence radius** and **convergence interval** of the Taylor series expansion of $f(x) = ArcTan(x)$ centered at $x = 2$. [10 pts]

6 Use the Ratio Test to find the convergence radius of the following power series:

$$1 - x + 2^2x^2 - 3^2x^3 + 4^2x^4 - 5^2x^5 + \ldots + (-1)^k k^2 x^k + \ldots$$

[10 pts]

Hint:you may need to take the limit of your ratio.

7 Set $Int(n) = \int_0^1 x^n e^{-x} dx$.
Find a formula that expresses $Int(n)$ in terms on $Int(n-1)$. [10 pts]

8 Solve the following differential equation with the initial condition $y(0) = 3$.

$$\frac{dy}{dx} = \frac{1}{y(x+1)}$$

[5 pts]

9 Compute the volume of the solid bounded above by the surface $z = e^x + x^2 y$ and below by the region R that lies between the graphs of the three functions $y = x$ and $y = 1/x$ and $y = 4x$. [10 pts]