

problem 1

Ten people, P1, P2, ..., P10, are permuted (lined up) at random. Find the probability that P2 and P3 will be separated by 3 people.

For instance here are a couple of permutations in which P2 and P3 are separated by 3 people.

P1 P2 P10 P9 P4 P3 P6 P5 P7 P8

P1 P4 P5 P6 P7 P3 P8 P9 P10 P2

solution 1

method 1

Since they are lined up *at random*, I'll use fav lineups/total lineups.

Total number of lineups is $10!$

For the fav:

step 1 pick 2 seats for P2 and P3

Can be done in 6 ways (could be seats 1,5; 2,6; ... ; 6,10)

step 2 Fill those 2 seats with P2 and P3. Can be done in 2 ways.

step 3 Fill the other seats. Can be done in $8!$ ways.

Answer is $\frac{6 \cdot 2 \cdot 8!}{10!}$ ($= \frac{2}{15}$)

method 2

(But I was not totally confident about this method until I saw that it agreed with the first method)

Forget about actually seating the people. The problem really amounts to choosing 2 seats (a *committee* of 2 seats) from the seats S1, ..., S10. We want the prob that the two seats that you draw are 3 apart.

The total number of ways in which 2 seats can be chosen is $\binom{10}{2}$

There are 6 fav committees of seats: S1S5, S2S6, S3S7, S4S8, S5S9, S6S10

The answer is

$$6 / \binom{10}{2}$$

Check that the two answers agree

$$\text{second answer} = \frac{6}{\binom{10}{2}} = 6 \cdot \frac{2! \cdot 8!}{10!} = \text{first answer} \quad \text{QED}$$