

## problem 2

Start with 18 men and 15 women.

Pick a committee of 5 men and 6 women.

Mary and John just got divorced so if they are both on the committee they will fight with each other.

Find the probability of a peaceful committee.

Try it twice

- (a) by switching to the opposite event
- (b) without switching

And show briefly how you get your answer.

## solution 2

$$\begin{aligned} \text{(a) } P(\text{peaceful}) &= 1 - P(\text{not peaceful}) \\ &= 1 - P(\text{J and M are both on the committee}) \end{aligned}$$

To find  $P(\text{J and M are both on the committee})$  use fav/total.

$$\text{Total} = \binom{18}{5} \binom{15}{6}$$

*Question*

Why isn't the total  $\binom{33}{11}$  ?

*Answer*

That total would go with the experiment "pick 11 people". But my experiment specifically picks 5 Men and 6 Women, not any old 11 people.

To count the fav:

step 1 Put J on the committee and pick 4 more men from the 17 other men

Can be done in  $\binom{17}{4}$  ways

step 2 Put M on the committee and pick 5 more from the 14 other women

Can be done in  $\binom{14}{5}$  ways.

$$\text{So fav} = \binom{17}{4} \binom{14}{5}$$

$$\text{Answer} = 1 - \frac{\binom{17}{4} \binom{14}{5}}{\binom{18}{5} \binom{15}{6}} \quad [ = 8/9 ]$$

(b) *method 1*

$$P(\text{peaceful}) = P(\text{J but not M} \quad \text{OR} \quad \text{M but not J} \quad \text{OR} \quad \text{neither})$$

The events

J but not M

M but not J

neither

are mutually exclusive so, by the OR rule,

$$P(\text{peaceful}) = P(\text{J but not M}) + P(\text{M but not J}) + P(\text{neither}).$$

Each one is fav/total

$$\text{Total} = \binom{18}{5} \binom{15}{6}$$

To get the fav for "J but not M", pick 4 more men from the non-J's, pick 6 women from the non-M's.

$$\text{Fav is } \binom{17}{4} \binom{14}{6}$$

$$\text{Similarly the fav for "M but not J" is } \binom{17}{5} \binom{14}{5}$$

To get the fav for "neither" pick 5 men from the 17 non-J's, pick 6 women from the 14 non-M's. Fav is  $\binom{17}{5} \binom{14}{6}$

$$\text{Answer is } \frac{\binom{17}{5} \binom{14}{6} + \binom{17}{4} \binom{14}{6} + \binom{17}{5} \binom{14}{6}}{\binom{18}{5} \binom{15}{6}}$$

*method 2*

$$P(\text{peaceful}) = P(\text{notJ or notM})$$

The events "not J" and "not M" are not mutually exclusive so when we use the OR rule there is a 2-at-a-time term.

$$P(\text{peaceful}) = P(\text{notJ}) + P(\text{notM}) - P(\text{notJ and notM})$$

To find the fav for "not J", pick 5 men from the 17 non-J's, pick 6 women from the 15 women.

To find the fav for "not M", pick 5 men from the 18 men and pick 6 women from the 14 non-M's

To find the fav for "neither", pick 5 men from the 17 non-J's and pick 6 women from the 14 non-M's.

$$\text{Answer is } \frac{\binom{17}{5} \binom{15}{6} + \binom{18}{5} \binom{14}{6} - \binom{17}{5} \binom{14}{6}}{\binom{18}{5} \binom{15}{6}}$$