

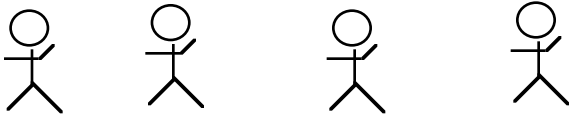
problem #3

Given four girls G1, ..., G4 and four boys B1, ..., B4

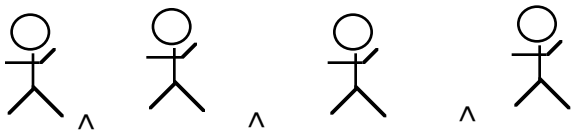
The idea is to count the ways of lining up so that no two girls are next to one another (i.e., keep the girls apart).

(a) Here's a wrong way to do it.

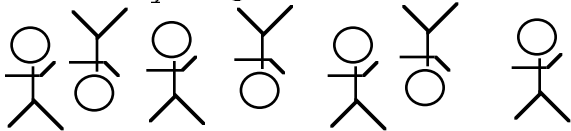
Line up the girls Can be done in $4!$ ways



Look at the 3 between.



Pick a boy to go in each between. Can be done in $4*3*2$ ways



Now there are 8 between/ends.

Pick one of them for the last boy. Can be done in 8 ways

Answer is $4! * 4 * 3 * 2 * 8$

Explain very concretely why it's wrong by finding two outcomes that are counted as different by this method but are really the same (so the wrong answer is an overcount).

(b) Get the right answer.

solution 3

(a) This answer overcounts. It counts the following as different outcomes when they are really the same

1. Line up girls G1 G4 G3 G2

Put B1 in the first between, B3 in the second between, B4 in the third between

So far I have G1 B1 G4 B3 G3 B4 G2

Put B2 in the second of the 8 between/ends

Outcome is G1 B2 B1 G4 B3 G3 B4 G2

2. Line up girls G1 G4 G3 G2

Put B2 in the first between, B3 in the second between, B4 in the third between

So far I have G1 B2 G4 B3 G3 B4 G2

Put B1 in the third of the 8 between/ends

Outcome is G1 B2 B1 G4 B3 G3 B4 G2

This method counted these two outcomes as different but they are the same lineup.

(b) Follow the standard method to keep the girls apart.

Step 1 Line up the others, i.e., the 4 boys. Can be done in $4!$ ways.

Step 2 Put the girls in the between/ends

G1 can pick any of the 5 between/ends

G2 can go in any of the remaining 4 between/ends

G3 can go in any of the remaining 3 between/ends

G4 can go in any of the remaining 2 between/ends

Can be done in $5*4*3*2$ ways

method 2 for step2

Pick 4 of the 5 between/ends. Can be done in $\binom{5}{4}$ ways.

Match the 4 girls to those 4 between/ends. Can be done in $4!$ ways

So step 2 can be done in $\binom{5}{4} 4!$ ways

Final answer is $4! * 5*4*3*2$

Or equivalently, $4! \binom{5}{4} 4!$