problem 3

First look at Section 1.3, top of page 4 (and problem 12) for keeping things together in a permutation.

Find the prob that a randomly-chosen permutation of the 26 letters of the alphabet contains FISH or RAT or BIRD

solution 3

P(fish or rat or bird)

= P(fish) + P(rat) + P(bird) − P(fish & rat)

There is only one 2-at-a-time term since a perm can't have both fish and bird (because they each need an i) and can't have both rat and bird (since they each need an r).
There is no 3-at-a-time term since a perm can't have fish and rat and bird.

To find all these probs, use fav/total.
The total number of perms is 26!

To count the fav for P(fish), permute 23 things, a FISH clump and the other 22 letters (don't permute within the clump since we want the letters in the clump in the order F I S H). Can be done in 23! ways.

Similarly for P(bird).

To find the fav for P(rat), permute 24 things, a RAT clump and the other 23 letters. Can be done in 24! ways.

To find the fav for P(fish & rat), permute 21 things, a RAT clump, a FISH clump and the other 19 letters.
Can be done in 21! ways.

Answer is $\frac{23! + 24! + 23! - 21!}{26!}$