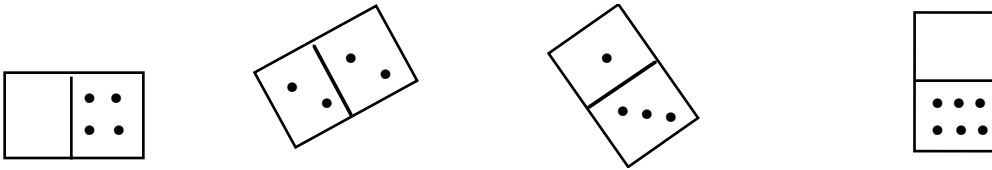


problem #4

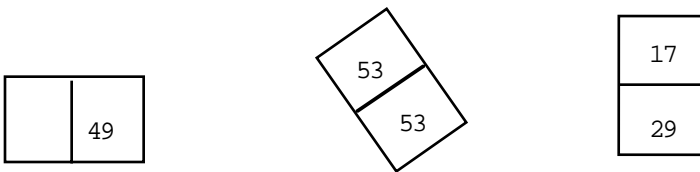
Here are some pictures of dominoes.



The face of a domino is divided in half and each half has either no dots, one dot, ..., six dots.

A box of dominoes contains every possible configuration once.

A super-domino is like an ordinary domino except that each half has anywhere from no dots up to 99 dots.



How many super-dominoes are there in a box? Show how you get your answer.

There are several methods that work.

solution 4

METHOD 1

Each domino is a committee of size 2 from a population of 100 (the numbers 0-99) with repetition allowed. Use the stars and bars formula.

Answer is $\binom{100 + 2 - 1}{2} = \binom{101}{2} \quad [= 5050]$

METHOD 2

I can write out the whole list in a systematic fashion and count it by inspection.

0 paired with 0,1,2,..., 99	100 of these
1 paired with 1,2,3,... 99 [but not with 0 since that has been counted already]	99 of these
2 paired with 2,3,4,..., 99 [but not with 0,1 since that has been counted already]	98 of these
...	
98 paired with 98, 99	2 of these
99 paired with 99	1 of these

Answer is $100 + 99 + 98 + 97 + \dots + 1$

METHOD 3

Pick committees of size 2 from the 100 numbers. Can be done in $\binom{100}{2}$ ways.

But these are ordinary committees, without repeated members, so this leaves out the dominoes in which the two sides have the same value. There are 100 of these (two 0's, two 1's, ..., two 99's).

So the total number of dominoes is $\binom{100}{2} + 100$

METHOD 4

100 * 100 counts this list:

- (1) 96 followed by 2
 - (2) 2 followed by 96
 - (3) 96 followed by 96
 - (4) blank followed by 6
- etc.

But this isn't counting dominoes because items (1) and (2) are the same domino so the domino [2 96] is being counted twice.

On the other hand dominoes like [96 96] get counted only once.

So the "answer" 100*100 isn't right. And it can't be fixed up by just dividing by 2. Here are three ways to fix it up.

method 4a

Only dominoes with a different number of dots on each side are counted twice. So subtract away the number of dominoes with a different number of dots on each side.

This number is $\binom{100}{2}$

Answer is $100 * 100 - \binom{100}{2}$

method 4b

Dominoes with two different sides are counted twice.

Dominoes with the same number of dots on each side (there are 100 such dominoes) are counted only once.

So subtract away 100, divide by 2 and then add the 100 back in again.

Answer is $(100*100 - 100)/2 + 100$

method 4c

Dominoes with two different sides are counted twice.

Dominoes with the same number of dots on each side are counted only once. There are 100 of these.

So before dividing 100*100 by 2, add in another 100 so that everyone is counted twice and then divide by 2.

Answer is $(100*100 + 100)/2$

METHOD 5 (incredibly clever)

Choose a committee of size 2 from a population of *,0,1,...,99 where the committee *,3 for instance represents the domino [3 3].

Answer is $\binom{101}{2}$