

**problem 5**

John's train will reach the station at some random time between 10:00 and 11:00. Mary's train will reach the station at some random time between 10:00 and 11:30.

John's train has a 10 minute layover but Mary's train rolls right through. Find the prob that John and Mary see each other at the station.

**solution 5**

Let's call 10:00 time 0 and measure time in minutes.

Let his train's arrival time be  $x$  and her train's arrival time be  $y$ . Then  $(x,y)$  is uniformly distributed in the rectangular region where  $0 \leq x \leq 60$ ,  $0 \leq y \leq 90$

$P(\text{they see each other})$

$= P(\text{J arrives first and then M arrives no more than 10 minutes later})$

$$= P(y \geq x \text{ and } y \leq x + 10)$$

Another way of looking at it is to see that M must be between J and J+10 so

$$P(\text{they meet}) = P(x \leq y \leq x + 10)$$

The universe is the rectangular region where  $0 \leq x \leq 60$  and  $0 \leq y \leq 90$ .

To get the fav region (Fig 1), first draw the line  $y = x$  and the line  $y = x + 10$

The fav region is that part of the universe above the line  $y = x$  and under the line  $y = x + 10$ .

The total area is  $60 \cdot 90 = 5400$

Fav area = total - unfav

$$= 5400 - \text{rectangle A} - \text{triangle B} - \text{triangle C} \quad (\text{Fig 2})$$

$$= 5400 - 20 \cdot 60 - \frac{1}{2} 60 \cdot 60 - \frac{1}{2} 60 \cdot 60$$

$$= 600$$

$$\text{Answer} = \frac{\text{fav}}{\text{total}} = \frac{600}{5400}$$

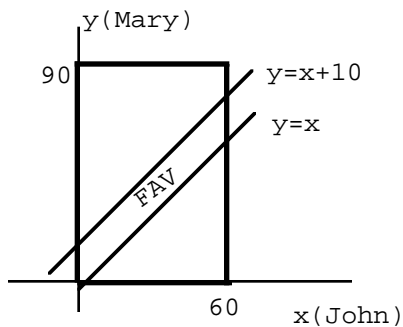


FIG 1

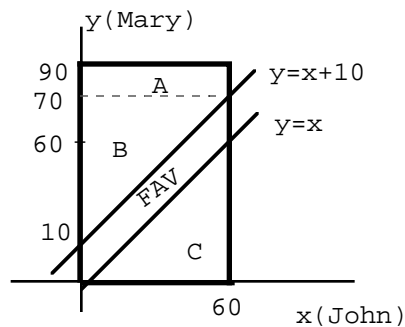


FIG 2