

problem #6

Go back to problem #5: How many permutations of APPLETREE have an E at the left end or the right end or in the middle spot.

And do it again, this time by switching to the opposite event.

solution 6

$$\text{total} = \frac{9!}{2! 3!}$$

Opp of "E at left end OR E at right end OR E middle"

is "E not at left end AND E not at right end AND E not middle "

To count the opp, pick 3 space from the non-ends-non-middle for the E's.

Can be done in $\binom{6}{3}$ ways

Then permute the remaining 6 letters (which include two P's) to go in the remaining six spots. Can be done in $\frac{6!}{2!}$ ways.

$$\text{So opp} = \binom{6}{3} \frac{6!}{2!}$$

$$\text{Answer is } \frac{9!}{2! 3!} - \binom{6}{3} \frac{6!}{2!} \quad [= 23040]$$