

problem 7

If A shoots at a target, the prob that she hits the target is .6

If B shoots at a target, the prob that she hits the target is .3

If C shoots at a target, the prob that she hits the target is .8

They all shoot once (independently of one another).

You inspect the target and find one arrow in it.

Find the probability that it was B that hit the target.

Indicate where you used the fact that they shoot independently.

solution 7

The problem says that the event "one arrow hit" has happened and asks about the event "B hit".

So the problem is to find the CONDITIONAL PROBABILITY $P(B \text{ hit} | \text{one arrow hit})$

$$P(B \text{ hit} | \text{one arrow hit}) = \frac{P(B \text{ hit and one arrow hit})}{P(\text{one arrow hit})}$$

$$\begin{aligned} \text{numerator} &= P(B \text{ hits and only one arrow hits}) \\ &= P(A \text{ misses and } B \text{ hits and } C \text{ misses}) \\ &= (.4)(.3)(.2) \quad \text{by the AND rule for independent events} \end{aligned}$$

$$\begin{aligned} \text{denominator} &= P(A \& \bar{B} \& \bar{C} \text{ or } \bar{A} \& B \& \bar{C} \text{ or } \bar{A} \& \bar{B} \& C) \\ &= P(A \& \bar{B} \& \bar{C}) + P(\bar{A} \& B \& \bar{C}) + P(\bar{A} \& \bar{B} \& C) \\ &\quad \text{by the OR rule for mutually exclusive events} \\ &= (.6)(.7)(.2) + (.4)(.3)(.2) + (.4)(.7)(.8) \\ &\quad \text{by the AND rule for ind events} \end{aligned}$$

$$\text{Answer is } \frac{(.3)(.4)(.2)}{(.6)(.7)(.2) + (.4)(.3)(.2) + (.4)(.7)(.8)} \quad \left[= \frac{6}{83} \right]$$