

**problem 9**

How many permutations of the alphabet contain neither fish nor rat nor bird, i.e., do not contain fish and do not contain rat and do not contain bird.

For example these are OK

ABCDEF GHI JKLMNOP QRSTUVWXYZ  
BACDEF GHI JKLMNOP QRSTUVWXYZ

but this one is not

AB FISH CDEWXYZGJKLMN OPQRTUV

**solution 9**

Use total - opposite  
Total = 26!

$$\begin{aligned} \text{opp} &= N(\text{fish or rat or bird}) \\ &= N(\text{fish}) + N(\text{rat}) + N(\text{bird}) - N(\text{fish \& rat}) \end{aligned}$$

There is only one 2-at-a-time term since a perm can't have both fish and bird (because they each need an i) and can't have both rat and bird (since they each need an r).  
There is no 3-at-a-time term since a perm can't have fish and rat and bird

To find N(fish), permute 23 things, a FISH clump and the other 22 letters (don't permute within the clump since we want the letters in the clump in the order FISH). Can be done in 23! ways.

Similarly for N(bird).

To find N(rat), permute 24 things, a RAT clump and the other 23 letters. Can be done in 24! ways.

To find N(fish & rat), permute 21 things, a RAT clump, a FISH clump and the other 19 letters. Can be done in 21! ways.

$$\text{opp} = 23! + 24! + 23! - 21!$$

Answer is  $26! - [ 23! + 24! + 23! - 21! ]$