**problem 10**

Here's a specific instance of something called Vandermonde's identity:

$$\binom{80}{0}\binom{20}{17} + \binom{80}{1}\binom{20}{16} + \binom{80}{2}\binom{20}{15} + \ldots + \binom{80}{17}\binom{20}{0} = \binom{100}{17}$$

footnote

The general version is $\sum_{k=0}^{r} \binom{n}{k}\binom{m}{r-k} = \binom{n+m}{r}$

In my specific instance, n = 80, m = 20, r = 17

Explain why it's true by making up a counting story. In other words, give a combinatorial proof.
(I don't care if you work with the specific instance or the general version)

**solution 10**

Start with 80 men and 20 women.
Look at committees of 17 people.
There are (100 on 17) of them.

On the other hand, the committees fall into these (mutually exclusive) cases.

0 M & 17 W   There are (80 on 0)(20 on 17) of these committees
1 M & 16 W   There are (80 on 1)(20 on 16) of these committees
2 M & 16 W   There are (80 on 2)(20 on 15) of these committees
.
.
.

17 M & 0 W   There are (80 on 17)(20 on 0) of them

The total number of committees is the sum of all these numbers of particular committees.
So

$$\binom{80}{0}\binom{20}{17} + \binom{80}{1}\binom{20}{16} + \binom{80}{2}\binom{20}{15} + \ldots + \binom{80}{17}\binom{20}{0} = \binom{100}{17}$$

QED