

**problem 12**

How many edges in  $K_n$  (the complete graph with  $n$  vertices)

**solution 12***method 1*

Each edge is a committee of size 2 from a population of the  $n$  vertices.

For example, the edge between vertex  $v_1$  and vertex  $v_7$  is the "committee"  $v_1, v_7$ .

Answer is  $\binom{n}{2}$

*method 2*

Use "left end of the edge" and "right end of the edge" as slots.

Left can be filled in  $n$  ways.

right can be filled in  $n-1$  ways.

But this double counts: it counts edge AB as different from edge BA.

It double counts precisely by a factor of 2. So the correct answer is  $\frac{n(n-1)}{2}$

*method 3*

From  $v_1$  there is an edge to each of  $v_2, \dots, v_n$

So far,  $n-1$  edges

From  $v_2$  there is an edge to each of  $v_3, \dots, v_n$

(don't count the  $v_1v_2$  edge -- it was counted already)

Another  $n-2$  edges

From  $v_3$  there is an edge to each of  $v_4, \dots, v_n$

(don't count edges to  $v_1$  and  $v_2$  -- they have been counted already)

Another  $n-3$  edges

etc.

Answer is  $n-1 + (n-2) + (n-3) + \dots + 1$