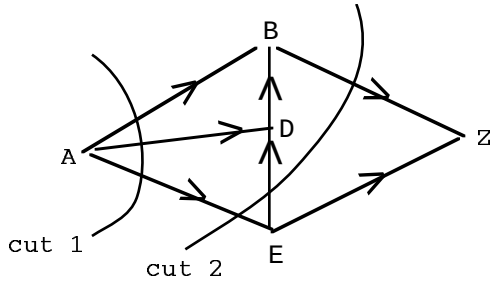
problem 15

Here is a network flow with two cuts.
The actual flows are not given.
Use the notation AB for the flow on edge AB, etc.
(The capacities will be irrelevant here.)
Find the flow across cut 1 (in the abstract) and find the flow across cut 2 (in the abstract) and then show that they are the same.

What you have to work with is conservation of flow at each vertex (except for the source and sink) because that's an axiom of network flows.
In other words, you're going to show that if flow is conserved at each vertex other than A and Z then it is the same across the two cuts.



KEEP READING FOR DIRE WARNING
To show that

cut 1 flow = cut 2 flow

do NOT write like this:

cut 1 flow = cut 2 flow

UGH ⋮ UGH

0 = 0
TRUE

Any "proof" in mathematics that *begins* with what you want to prove and *ends* with TRUE is at best badly written and at worst incorrect and *drives me crazy*.

With this "method" I can prove that 3 = 4:

```
3 = 4  (start with what you want to prove)
4 = 3  (just reverse the preceding line)
7 = 7     (add the two preceding lines)
 TRUE
 So conclude  that 3 = 4  (??????)
```

To show that cut 1 flow = cut 2 flow arrange your work like this:

```
cut 1 flow = ...
           = ...
           = ...
           = cut 2 flow
```

Or like this:

```
cut 1 flow = ...
           = ...
           = xyz

cut 2 flow = ...
           = ...
           = xyz
```

Therefore cut 1 flow = cut 2 flow

Or like this:

```
    cut 1 flow − cut 2 flow = ...
                            = ...
                            = ...
                            = 0

      Therefore cut 1 flow = cut 2 flow
```

solution #15

I'm using the notation AB for the flow on edge AB.

cut 1 flow = AB + AD + AE

cut 2 flow = AE − ED + BZ

By conservation of flow at vertex B,

(1) AB = BZ − DB

By conservation of flow at vertex D,

(2) AD = BD − ED

So

```
   cut 1 flow =  BZ − BD + BD − ED + AE

              = BZ − ED + AE
```

So cut 1 flow = cut 2 flow QED

footnote
For any third cut, you could similarly show that cut 3 flow = cut 1 flow (cut 1 is special because it doesn't "overlap" any other cut, i.e., the rivers don't cross). That shows that cut 2 flow = cut 3 flow and more generally that all cut flows are the same.