problem 17

Dominoes again.

An n–domino is like an ordinary domino except that each half-face can contain anywhere from no dots up to n dots. Ordinary dominoes are 6-dominoes.
(Problem #4 was about 99-dominoes.)

The contents of a box of n–dominoes and a box of m-dominoes will overlap.
For example, the domino in Fig 1 would be found in a box of 49-dominoes and also found in a box of 50-dominoes and in general in every box of n-dominoes where $n \geq 49$.
The domino in Fig 2 is a 29-domino and also a 30-domino and in general an n-domino where $n \geq 29$.

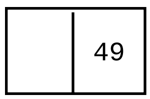

FIG 1

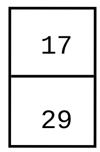

FIG 2

Let y_n be the number of n-dominoes. For instance, y_6 is the number of ordinary dominoes.
In problem 6, you found y_{99}, the number of 99-dominoes.
The idea here is to be able to find y_n for various n's with a different approach than that of problem 6.

(a) Write a recursion relation satisfied by the sequence y_n.
And find IC (the appropriate number to go with your recursion relation).
And explain briefly.

(b) If you got part (a) then use it to find the number of ordinary dominoes.

solution 17

(a) Every (n–1)-domino is also an n-domino. And in addition there are the following new n-dominoes:

[0,n]
[1,n]

⋮

[n,n]

There are n+1 new ones. So $y_n = y_{n-1} + n+1$
This is a first order recursion relation. We need one IC

There is just one 0-domino, namely [0 0]. So as an IC use $y_0 = 1$.

(b) $y_1 = y_0 + 2 = 1 + 2 = 3$
$y_2 = y_1 + 3 = 3 + 3 = 6$
$y_3 = y_2 + 4 = 6 + 4 = 10$
$y_4 = y_3 + 5 = 10 + 5 = 15$
$y_5 = y_4 + 6 = 15 + 6 = 21$
$y_6 = y_5 + 7 = 21 + 7 = 28$