problem 19

Look at the difference equation

$$y_{n+2} - y_{n+1} - 6y_n = 0$$

with IC

$$y_0 = 1,\ y_1 = 4.$$

(a) Before doing any solving, use iteration to find y_3.

(b) Now solve and find a formula for y_n.

(c) Use the formula from part (b) to find y_3 again, as a check.

solution 19

(a) $y_{n+2} = y_{n+1} + 6y_n$ so

$$y_2 = y_1 + 6y_0 = 4 + 6\cdot 1 = 10$$

$$y_3 = y_2 + 6y_1 = 10 + 6\cdot 4 = 34$$

(b) $\lambda^2 - \lambda - 6 = 0$

$(\lambda-3)(\lambda+2) = 0$

$\lambda = 3, -2$

General sol is $y_n = A\ 3^n + B(-2)^n$

To satisfy the IC we need

$1 = A + B$

$4 = 3A - 2B$

Solve to get $A = \frac{6}{5}$, $B = -\frac{1}{5}$.

Sol is $y_n = \frac{6}{5}\cdot 3^n - \frac{1}{5}(-2)^n$

(c) $y_3 = \frac{6}{5}\cdot 3^3 - \frac{1}{5}(-2)^3 = 34$