problem 20

In problem #18, u_n was the number of words of length n in which an A doesn't appear until after a B. And u_n satisfied the recurrence relation

$$u_n = 24u_{n-1} + 26^{n-1}$$

with IC $u_1 = 25$.

(a) Rewrite the rr so that it starts with u_{n+1} instead of u_n

(b) Solve the difference equation. You can use either the original or the rewritten version. Should get the same solution either way.

solution 20

(a) $u_{n+1} - 24u_n = 26^n$ with IC $u_1 = 25$.

(b) *version 1* Using $u_{n+1} - 24u_n = 26^n$

First find the homog solution, i.e., the sol to $u_{n+1} - 24u_n = 0$.

The characteristic equation is $\lambda - 24 = 0$

So $\lambda = 24$, $h_n = A24^n$

Then try $p_n = B\ 26^n$. Need

$$B\ 26^{n+1} - 24\ B26^n = 26^n$$

$$26B\ 26^n - 24B\ 26^n = 26^n$$

Equate coeffs of 26^n: $26B - 24B = 1$, $B = 1/2$

$$p_n = \frac{1}{2}\ 26^n$$

Gen $u_n = h_n + p_n = A24^n + \frac{1}{2}\ 26^n$

Now use the IC to find A.

Need $25 = 24A + \frac{1}{2}\ 26$

$A = 1/2$

The final solution is $u_n = \frac{1}{2}\ 24^n + \frac{1}{2}\ 26^n$