problem 21

In problem 17, y_n was the number of n-dominoes. And y_n satisfied the rr

$y_n = y_{n-1} + n+1$

with IC

$y_0 = 1.$

(a) Solve to find a formula for y_n.

(b) Check that when you use the formula to find y_{99} you get the same answer [5050] that I got back in problem 4.

solution 21

(a) I'm going to use this version: $y_{n+1} - y_n = n+2.$

Start by solving the characteristic equation $\lambda - 1 = 0$. Then $\lambda = 1$, $h_n = A\ 1^n = A$. Ordinarily I would try $p_n = Bn + C$ but C is a homog solution so I have to step up and try $p_n = n(Bn + C) = Bn^2 + Cn$.

Substitute this trial p_n in the rr and find B and C so that it works.

Need $B(n+1)^2 + C(n+1) - Bn^2 - Cn = n+2.$

Equate n^2 coeffs: Happens automatically since the n^2's cancel out on the left.

Equate n coeffs: $2B = 1$, $B = 1/2$.

Equate constant terms: $B + C = 2$, $C = 3/2$.

So $p_n = \frac{1}{2}n^2 + \frac{3}{2}n$ and the general solution is $y_n = h_{nn} + p_n = A + \frac{1}{2}n^2 + \frac{3}{2}n.$

To satisfy the IC $y_1 = 3$, I need $3 = A + \frac{1}{2} + \frac{3}{2}$, $A = 1$.

Solution is $y_n = 1 + \frac{1}{2}n^2 + \frac{3}{2}n.$

footnote

By the way, here's what happens if you try $p_n = Bn + C$ with*out* stepping up. You need

$$B(n+1) + C - (Bn+C) = n+2$$

But you can't make this be true. The coeff of the n term on the right is 1 but the coeff of the n term on the left is 0 (i.e., there is no n term on the left, it cancels out)

This means that there is no solution of the form Bn + C.

(b) $y_{99} = 1 + \frac{1}{2}(99)^2 + \frac{3}{2} * 99 = 5050.$