

**MATH 104 — FALL 2004 MIDTERM EXAM
INTRODUCTION TO ANALYSIS**

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF CALIFORNIA, BERKELEY
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NAME: _____

ID NUMBER: _____

- (1) Do not open this exam until you are told to begin.
- (2) This exam has 7 pages including this cover. There are 6 questions total. You have 80 minutes.
- (3) No aids such as calculators, notes or books are permitted.
- (4) You will be handed scrap paper for use.
- (5) Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to the proctor when you turn in the exam.
- (6) You may use any results proved in class. However, you must include complete hypotheses and conclusions to ensure credit.
- (7) Please turn off all cell phones.

PROBLEM	POINTS	SCORE
1	8	
2	8	
3	8	
4	8	
5	8	
6	10	
TOTAL	50	

1. Give precise statements of
 - (a) completeness of \mathbb{R} ;
 - (b) a metric space;
 - (c) Extreme value theorem for (arbitrary) metric spaces.

2. State and prove the intermediate value theorem.

3. Let (M, d_1) and (N, d_2) be two metric spaces and $f : M \rightarrow N$ be a continuous function. Let $S \subset M$ be a closed and let \tilde{S} be the *graph* of f , i.e.,

$$\tilde{S} = \{(s, f(s)) : s \in S\} \subseteq M \times N.$$

Prove that \tilde{S} is closed.

4. Let $\{a_i\} \rightarrow a$ and $\{b_i\} \rightarrow b$ be two convergent sequences in \mathbb{R} with $a_i \leq b_i$ for each i . Prove $a \leq b$.

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q} \\ x^2 & \text{otherwise.} \end{cases}$$

Is f continuous at $x = 0$? Give your answer with a proof.

6. Let (M, d) be a compact metric space. Let $f : M \rightarrow M$ be continuous and satisfying $d(f(x), f(y)) \leq \frac{1}{2}d(x, y)$ for any $x, y \in M$. Prove that f has a *fixed point*, i.e., a point $x \in M$ such that $f(x) = x$. Hint: construct a useful sequence to consider.